Sample Exam Solutions

1. True
2. False
3. True
4. False
5. True
6. True
7. True
8. True
9. True
10. False

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(1) A: 1^{st} makes profit $P(A) = 7/10$

B: 2^{nd} doesn't make profit. $P(A|B) = 3/5$

$P(A \text{ and } B) = 1/5$.

$P(2^{rd} \text{ makes profit}) = P(\bar{B}) = 1 - P(B)$.

$P(B) = ?$ $\Rightarrow$ $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)}$

$\Rightarrow P(B) = P(A \text{ and } B) / P(A|B) = (1/5) / (3/5) = \boxed{\frac{1}{3}}$

So $P(\bar{B}) = 1 - \frac{1}{3} = \boxed{\frac{2}{3}}$

2 a) $P(X > 5) = \boxed{1}$ (all prizes are more than \$5).

b) First need $P(X=20)$

$$P(X=20) = 1 - [.8 + .08 + .001] = \boxed{0.119}$$

$$\mu = \sum x p(x)$$

$$= 10(.8) + 20(.119) + 100(.08) + 1000(.001)$$

$$= \boxed{19.38}$$

$$\sigma^2 = \sum x^2 p(x) - \mu^2$$

$$= 10^2(.8) + \dots + 1000^2(.001) - (19.38)^2$$

$$= 1552.02$$

$$\sigma = \sqrt{1552.02} = \boxed{39.40}$$

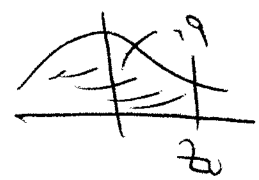
(3) X = balance. $X \sim N(1000, 240)$.

(a) $P(X < 1072) = P\left(Z < \frac{1072-1000}{240}\right)$

$= P(Z < 0.3) = \boxed{0.6179}$

(b) $P(X < x_0) = 0.9$. find x_0.

(i) $P(Z < z_0) = 0.9$



$\boxed{z_0 = 1.28}$ or 1.29

(ii) $1.28 = \frac{x_0 - 1000}{240}$

$x_0 = 240(1.28) + 1000 = \boxed{1307.20}$

(c) $P(\bar{X} < 1072) = P\left(Z < \frac{1072-1000}{240/\sqrt{64}}\right)$

$= P(Z < 2.4) = \boxed{0.9918}$

(4) a) $H_0: p = 0.5$

$H_1: p > 0.5$

$\hat{p} = \frac{146}{250} = 0.584$

$$z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{.584 - .5}{\sqrt{\frac{.5(1-.5)}{250}}} = 2.66$$

$\alpha = 0.01$: Reject H_0 if $z > z_{.01} = 2.33$.

Since $2.66 > 2.33$, reject H_0.

Brand A s often by more than 50% of people.

(b) p-value $= P(z > 2.66) = 1 - .9961 = \boxed{.0039}$

(c) (i) Random sample from population.

(ii) $np \geq 5$ and $n(1-p) \geq 5$

(5) Athletes: $n_1 = 55$, $\bar{x}_1 = 20.6$, $s_1 = 5.3$

NonAth: $n_2 = 200$, $\bar{x}_2 = 23.5$, $s_2 = 4.1$

$H_0: \mu_1 = \mu_2$ $\qquad$ $H_1: \mu_1 > \mu_2$

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{20.6 - 23.5}{\sqrt{\frac{5.3^2}{55} + \frac{4.1^2}{200}}} = \boxed{-3.76}$$

$\alpha = .05$: Reject H_0 if $z > z_{.05} = 1.645$ (or 1.64 or 1.65)

Since $-3.76 < 1.645$, don't reject H_0.

Mean hours expended by nonathletes is not smaller.

6 a $y' = a + bx$

$SS_{xy} = \Sigma xy - (\Sigma x)(\Sigma y)/n$

$= 703 - 22(247)/8$

$= 23.75$

$SS_{xx} = 10.5$ (given)

$b = SS_{xy}/SS_{xx} = 23.75/10.5 = 2.26$

$\bar{y} = 247/8 = 30.875$, $\bar{x} = 22/8 = 2.75$

$a = \bar{y} - b\bar{x} = 30.875 - 2.26(2.75) = 24.66$

$\boxed{y' = 24.66 + 2.26x}$

b $x = 2.5$: $y' = 24.66 + 2.26(2.5) = 30.31$

residual: $y - y' = 28 - 30.31 = -2.31$

c NOT COVERED BY US THIS TERM.

d $r = \dfrac{SS_{xy}}{\sqrt{SS_{xx}\,SS_{yy}}} = \dfrac{23.75}{\sqrt{(10.5)(112)}}$

$= 0.69$

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7 a $n=20$, $\bar{x}=63$, $s=7.65$

90% CI for μ: $\alpha=.1$, $df=(n-1)=19$.

$$t_{.05}=1.729$$

$$\bar{x}\pm t_{\alpha/2}\frac{s}{\sqrt{n}}=63\pm 1.729\left(\frac{7.65}{\sqrt{20}}\right)$$

$$=\boxed{(60.04,\ 65.96)}$$

b $\sigma=\sqrt{400}=20$, $E=2$

95% CI: $\alpha=.05$, $z_{\alpha/2}=1.96$

$$n=\left(\frac{z_{\alpha/2}\cdot\sigma}{E}\right)^2$$

$$=\left[\frac{(1.96)(20)}{2}\right]^2=384.16$$

so $\boxed{n=385}$