Statistics 2500-001 **Midterm** **Oct. 22, 2009**

SOLUTIONS

Last Name, First Name

30 Points

ID

YOUR TEST MUST BE SUBMITTED BY 4:45PM

- No books or notes permitted. A calculator is permitted.
- A formula sheet and normal distribution table are provided. Please **do not write on the formula sheet**, and return it with your test.
- Show your work and answers **clearly** in the space provided.
- This test has **6** questions on **5** pages.
- This test is marked out of **30** points. The point value of each question is indicated.

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(3) 1. A lottery offers one \$500 prize, one \$100 prize and four \$50 prizes. Five hundred tickets are sold at \$2 each. What is the expected profit for someone that buys one ticket?

profit = win − 2

= 498 with prob. $\frac{1}{500}$

= 98 " " $\frac{1}{500}$

= 48 " " $\frac{4}{500}$

= −2 " " $\frac{494}{500}$

x	498	98	48	−2
$p(x)$	$\frac{1}{500}$	$\frac{1}{500}$	$\frac{4}{500}$	$\frac{494}{500}$

$$\mu = \sum x p(x) = \frac{498 + 98 + 4(48) - 2(494)}{500} = \boxed{-\frac{2}{5}}$$

(1) 2. Give a list of 3 numbers from 1 to 10 (they don't all have to be different) that gives the smallest possible sample variance.

Choose all 3 same: (1,1,1), (4,4,4), etc.

Then $s^2 = 0$

3. To investigate preschooler's satisfaction with its product, a peanut butter company randomly selected 2 streets in a city, then got every preschooler on these 2 streets to rate the peanut butter on a 1–100 scale (the higher the score, the better the peanut butter). The ratings are below:

22 36 39 40 44 50 51 51 53 56 56 58
59 60 61 62 62 64 65 68 71 73 78 78

(1) (a) Is this an example of simple random sampling, stratified random sampling, cluster random sampling or systematic random sampling?

cluster random sampling (1)

(3) (b) Construct a stem-and-leaf plot of the data. Describe the shape of the distribution. Would you expect the the mean to be less than, greater than, or approximately equal to the median? Explain.

2	2
3	6 9
4	0 4
5	0 1 1 3 6 6 8 9
6	0 1 2 2 4 5 8
7	1 3 8 8

2|2 means 22

(2)

slightly skewed left. mean < median

(0.5) (0.5)

(2) (c) Find the mean of the peanut butter ratings.

$$\bar{X} = \frac{22 + 36 + \cdots + 78}{24}$$ (1)

$$= \frac{1357}{24} = 56.54$$ (1)

4. Salaries for experienced computer programmers are normally distributed with an average of \$64,800 per year and standard deviation is \$7200.

(3) (a) What is the probability that a randomly selected experienced programmer makes between \$66,000 and \$75,000 per year?

X = Salary $\quad X \sim N(64800, 7200)$

$P(66000 < X < 75000)$

$= P\left(\frac{66000-64800}{7200} < Z < \frac{75000-64800}{7200}\right)$

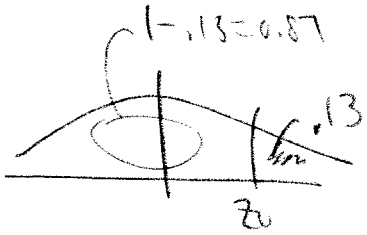
$= P(0.17 < Z < 1.42)$ ② for Z's

$= 0.9222 - 0.5675$ (Table E-2)

$= \boxed{0.3547}$

(4) (b) Above what salary would we find 13% of experienced programmers?

$P(X > X_0) = 0.13$ Find X_0.



Step 1: $P(Z > Z_0) = 0.13$

Then $P(Z < Z_0) = 0.87$ -①

Find 0.87 in MIDDLE of table $\quad Z_0 = 1.13$ (or 1.12) ①

Step 2: $Z = \frac{X-\mu}{\sigma}$

$1.13 = \frac{X_0 - 64800}{7200}$ ①

$X_0 = 1.13(7200) + 64800 = \boxed{72936}$ ①

5. My daughter's third birthday is tomorrow (you can send her a Happy Birthday email at *idontlikestats@hotmail.com*). She says that she wants an ice cream cake for her birthday. In fact, she claimed that 58% of preschool kids want an ice cream cake on their birthday. To check on this, we selected a random sample of 13 preschool kids to see if they wanted an ice cream cake.

(3) (a) What is the probability that at most 12 of the preschool kids want an ice cream cake on their birthday?

X = # want ice cream cake. $X \sim bin(13, .58)$

$$P(X \le 12) = 1 - P(X > 12)$$

$$= 1 - p(13)$$

$$= 1 - \left[\frac{13!}{13!(13-13)!} (.58)^{13} (1-.58)^{13-13} \right] \quad (2)$$

$$= 1 - (.58)^{13} = 1 - .0008$$

$$= \boxed{0.9992} \quad (-1)$$

(3) (b) What is the probability that less than 2 of the preschool kids want an ice cream cake on their birthday?

B-1

$$P(X < 2) = P(X=0) + P(X=1)$$

$$= \frac{13!}{0!(13-0)!} (.58)^0 (1-.58)^{13-0} + \frac{13!}{1!(13-1)!} (.58)^1 (1-.58)^{13-1} \quad (2)$$

$$= (.42)^{13} + 13(.58)(.42)^{12}$$

$$(1) = 1.26 \times 10^{-5} + .000227$$

$$= \boxed{.00024}$$

6. The medal counts from the 2006 Winter Olympics are below:

	Gold	Silver	Bronze
Germany	11	12	6
U.S.	9	9	7
Austria	9	7	7
Russia	8	6	8
Canada	7	10	7
Other	40	40	49

Define:

A: {medal is not gold}; B: {medal is U.S.};
C: {medal is for Canada or Russia, and is not silver}

(2) (a) Find $P(A \text{ or } B)$.

$P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$ (1)

$= \frac{168}{252} + \frac{25}{252} - \frac{16}{252} = \frac{177}{252} = 0.70$ (1)

(3) (b) Find $P(\bar{B} \text{ and } C)$.

Using $P(\bar{B} \text{ and } C) = P(\bar{B}) \cdot P(C)$ (1)

$P(\bar{B} \text{ and } C) = \frac{8+7+8+7}{252}$ (2)

$= \frac{30}{252} = 0.119$ (1)

(2) (c) Show whether or not A and B are independent.

Independent if: $P(A \cap B) = P(A) \cdot P(B)$

$P(A \text{ and } B) = \frac{9+7}{252} = \frac{16}{252}$ (1)

$P(A) \cdot P(B) = \left(\frac{168}{252}\right)\left(\frac{25}{252}\right) \neq \frac{16}{252}$ (1) Not independent