Testing Difference Between Two Means: Dependent Groups

Text: Sect. 9–5

EX 9.3: Nine men are randomly chosen to test a new weight loss diet.

Subject	1	2	3	4	5	6	7	8	9
Weight Before Diet	173	186	205	168	188	202	190	216	172
Weight After Diet	172	183	207	158	191	193	181	210	174

Is the diet effective?

It looks like we want to compare 2 groups.

So, use previous test that compares two population means?

PROBLEM: The observations are not independent (2 measurements on same person would be related).

- Sometimes called **paired data**: 2 observations per subject.

With paired data, we use a test based on the **differences** between the 2 sets of values.

Population mean of differences: μ_D
Sample mean of differences: $\bar{D}$
Sample standard deviation of differences: s_D
Sample size of differences: n_D

Test Procedure (p. 424, 425)

$\mathbf{H}_o$: $\mu_D = 0$

$\mathbf{H}_1$: 3 usual possibilities

Test statistic:

$$t = \frac{\bar{D}}{s_D/\sqrt{n_D}}$$

Rejection region, assumptions:
Identical to one-sample t-test about μ, using $(n_D - 1)$ df.

EX 9.3 (cont'd): Test at $\alpha = 0.10$ if diet is effective.

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      Before            173           ...                 172

      After             172           ...                 174
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 Diffs (Bef. - Aft.)      1   3  -2  10  -3   9   9   6  -2
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OMIT: Sections 9–3, 9–6 in text.

Test for Goodness of Fit

Text: Sect. 11–1, 11–2

NOTE: The lecture notes will use more mathematical notation than the text does on this topic.

EX 11.1: A study wishes to determine if expectant mothers go into labour at a constant rate during the day, or if they are more likely to go into labour at a particular time of day. The time of onset of labour was recorded for 1186 randomly selected pregnancies:

Time	Number
midnight–6am	417
6am–noon	287
noon–6pm	184
6pm–midnight	298

Is there evidence to suggest that the expectant mothers do not go into labour at a constant rate throughout the day?

General Form of Problem

Data fall into k different categories, with
p_i = P(falling in category i), $i = 1, \ldots, k$.

Category	1	2	...	k
Number	O_1	O_2	...	O_k
Prob.	p_1	p_2	...	p_k

Total of $n = (O_1 + \ldots + O_k)$ observations.

Also: $(p_1 + \ldots + p_k) = 1$.

EX. 11.1 (cont'd): $k = 4$
$O_1 = 417$, $O_2 = 287$, $O_3 = 184$, $O_4 = 298$, $n = 1186$.
p_1 = P(labour between midnight–6am), etc.

Goal: Do inference (make a *claim*) on p_i values.

In our example, suppose $p_1 = 1/4$.

How many of the 1186 mothers would we expect to find in the midnight–6am category?

How does this compare with the 417 we observe from midnight–6am? We need a test statistic to compare them.

Test Procedure (p. 503, 504)

$\mathbf{H}_o$: $p_1 = p_{1,o}, \ldots, p_k = p_{k,o}$

$\mathbf{H}_1$: H_o not true.

Test Statistic:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

where $E_i = np_{i,o}$ (**expected values**).

NOTE: χ: Greek letter **chi** (pronounced **Ki**).

Rejection Region: Reject H_o if $\chi^2 > \chi^2_\alpha$ using χ^2 **distribution** with $(k-1)$ df.

Assumptions:

1. Random sample drawn from population of interest.

2. Large sample size: $E_i \geq 5$ for every location (*cell*) in table.

χ^2 **Table**: Table G, p. 667. Also on course website.

EX 11.1 (cont'd): Assess the claim using $\alpha = 0.01$.

OMIT: Section 11–3 in text.