From the standard normal distribution, we know how to calculate the three kinds of probabilities: for given numbers a and b, where $a < b$,

- $P(z < b)$
- $P(z > a)$
- $P(a < z < b)$

Question: How can we find probabilities based on a normal distribution with $\mu \neq 0$ and $\sigma \neq 1$?

To deal with this problem, we introduce a transformation from a general normality $N(\mu, \sigma)$ to the standard normal distribution $N(0, 1)$.

$$\begin{array}{lcccc}
X & \longrightarrow & X-\mu & \longrightarrow & \frac{X-\mu}{\sigma} \\
\mu & \longrightarrow & 0 & \longrightarrow & 0 \\
\mu > 0, \text{shifts to left} & & & \mu < 0, \text{shifts to right} & \\
\sigma & \longrightarrow & \sigma & \longrightarrow & 1 \\
\sigma > 1, \text{be flatter} & & & \sigma < 1, \text{be steeper} &
\end{array}$$

Standard Score (z-score) (p.252): Indicates the number of standard deviations that a value is from (above or below) μ:

$$z = \frac{X - \mu}{\sigma}.$$

Why is this helpful?

RULE: If $X \sim N(\mu, \sigma)$, then

$$z = \frac{X - \mu}{\sigma} \sim N(0, 1).$$

Example 1: To qualify for security officer's training, recruits are tested for stress tolerance. The scores are normally distributed with mean of 62 and standard deviation of 8.

(a) What is the probability that a score falls below 63? Let X denote the score.

(b) What is the probability that a score falls above 60?

(c) What is the probability that a score falls between 60 and 63?

Example 2:(p. 265) To qualify for a police academy, candidates must score in the top 10% on a general abilities test. The test has a mean of 200 and a standard deviation of 20. Find the lowest possible score to qualify. Assume the test scores are normally distributed.

Example 3: The average age of Amtrack passengers train cars is 19.5 years. If the distribution of ages is normal and 20% of the cars are younger than 16 years, find the standard deviation.

Exercise: If a one-person household spends an average of \$40 per week on groceries, find the maximum and minimum dollar amounts spent per week for the middle 50% of one-person households. Assume that the standard deviation is \$5 and the variable is normally distributed. (Hints: Let a denote the minimum and b denote the maximum, the middle 50% means $P(X > b) = 0.25$ and $P(X < a) = 0.25$.)