Large Sample (n>30) Test and Confidence Interval (CI) for Mean

Formula:

100(1-α)% CI for mean

CI: $\bar{x} \pm Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$ (σ known) or $\bar{x} \pm Z_{\alpha/2} \frac{s}{\sqrt{n}}$ (σ unknown)

Test Statistic:

For testing, $H_0: \mu = \mu_0$ vs $H_1: \mu \neq \mu_0$

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} \ (\sigma \text{ known}) \text{ or } Z = \frac{\bar{x} - \mu_0}{s / \sqrt{n}} \ (\sigma \text{ unknown})$$

Where Z has N(0,1) distribution.

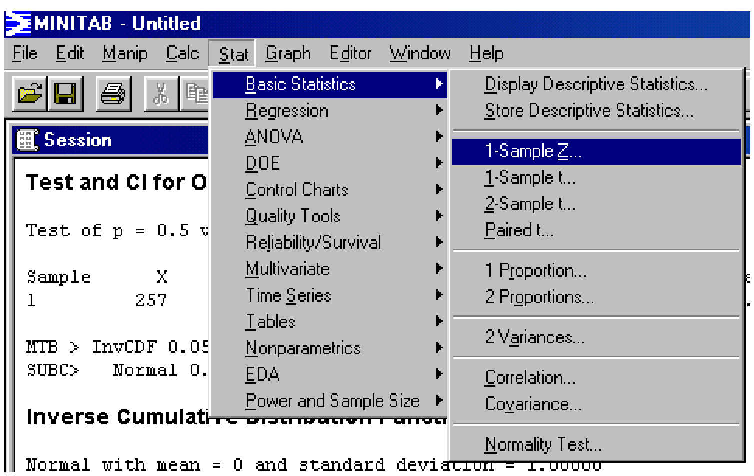
Example-1:

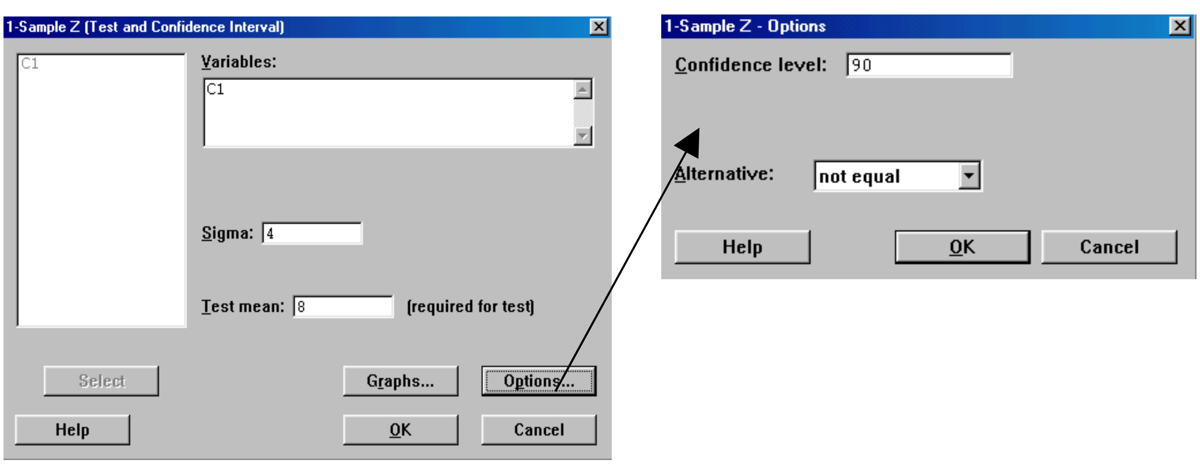
Suppose you have a sample of 225 observations (given in the 1st column of minitab worksheet) of which the ***known variance*** is 16 and you want to test:

$H_0: \mu = 8$ vs $H_1: \mu \neq 8$

Also you want find 90% CI for μ

Calculation in Mintitab





OUTPUT

```
Test of mu = 8 vs mu not = 8
The assumed sigma = 4

Variable          N      Mean     StDev   SE Mean
C1              225    11.565     4.010     0.267
Variable             90.0% CI            Z       P
C1            (  11.127,  12.004)    13.37   0.000
```

Small Sample (n<30) Test and Confidence Interval (CI) for Mean

Formula:

100(1-α)% CI for mean

CI: $\bar{x} \pm t_{\alpha/2}\left(\frac{s}{\sqrt{n}}\right)$ where $t_{\alpha/2}$ is based on (n-1) degrees of freedom (df).

Test Statistic:

For testing, H_0: $\mu = \mu_0$ vs H_1: $\mu \neq \mu_0$

$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$, where t has student's t-distribution with (n-1) df.

Example-2:
Given a sample
14, 9, 12, 14, 10, 9, 10, 12, 11, 9, 6, 2, 14, 10, 11, 12, 9, 6, 4, 8

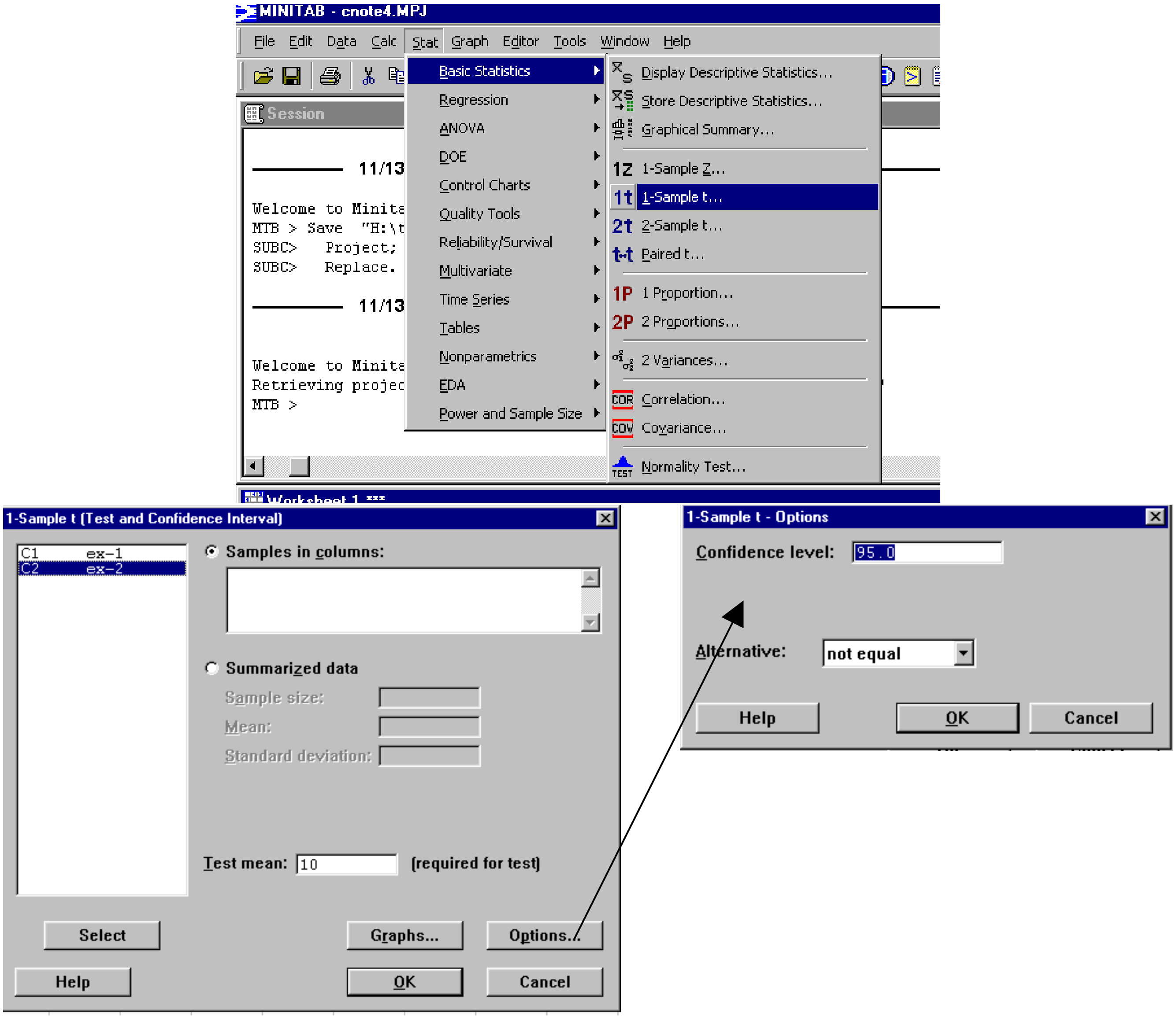
i) Test a) $H_0 : \mu = 10$ vs $H_1 : \mu \neq 10$
b) $H_0 : \mu = 10$ vs $H_1 : \mu > 10$
c) $H_0 : \mu = 10$ vs $H_1 : \mu < 10$

at 5% level of significance, 10% level of significance

ii) Find 95% confidence interval for μ

Solution:
Note that, here the sample size n = 20 (<30), i.e. we have a small sample and also population standard deviation σ is unknown. So we have to use t-distribution to perform the test and to find the confidence interval after estimating σ by the sample standard deviation S.

Calculation in Mintitab
(showing i(a), ii)



OUTPUT

One-Sample T: ex-2

```
Test of mu = 10 vs not = 10

Variable    N     Mean    StDev  SE Mean          95% CI               T      P
ex-2       20  9.65000  3.21632  0.71919  (8.14471, 11.15529)  -0.49  0.632
```

In the output Z (=13.37) for Example-1, and T (= -0.49) for Example-2 are the values of the corresponding test statistics and 'P' is the p-value which is 0.000 and 0.632 for Ex:1 and Ex:2 respectively.

If the p-value is less than or equal to your level (α), you can reject H_0.