SOLUTIONS

50 Points

Statistics 2500, Section 001
Assignment #3: Week of Oct. 26, 2009
Due on or before: Thursday, Nov. 19, 2009: 5pm

- The use of Minitab is **required** in the questions indicated. If you think you can use it in other questions, feel free to do so.
- Please **staple** the pages of your assignment together.
- Write your **name**, **ID number**, **lab instructor's name** and **day and time** you attend lab on your assignment.
- Assignments are to be passed into the assignment boxes located just to the left of the math/stats department's general office (HH–3003). Please put your assignment in the box that has your lab instructor's name on it:

  Melissa (Mon. 9am, 10:30am): **BOX 1**
  Vineetha (Mon. 3:30pm, Tues., Thurs. 1:50pm): **BOX 2**
  Chithran (Wed. 9am, 10:30am): **BOX 3**
  Hubert (Fri. 9am, 10:30am): **BOX 4**
  Yunqi (Fri. 2pm): **BOX 5**

- All problem numbers are taken from the textbook *Elementary Statistics* by Bluman and Mayer.

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1. A survey found that 64% of people under age 18 owned an iPod. A random sample of 277 people under age 18 is selected.
   (a) **Use Minitab** to find the probability that more than 160, but less than 180, people under age 18 owned an iPod. **PASS IN THE RELEVANT MINITAB OUTPUT**.
   (b) **Use Minitab** to find the probability that at least 156 people under age 18 owned an iPod. **PASS IN THE RELEVANT MINITAB OUTPUT**.

2. The number of loaves of white bread demanded daily at a bakery is normally distributed with mean 6800 loaves and variance 84000. The company decides to produce a sufficient number of loaves so that it will fully supply demand on 95% of all days.
   (a) How many loaves of bread should the company produce?
   (b) Based on (a), on what percentage of days will the company be left with more than 500 loaves of unsold bread?

3. A brand of spark plugs has an average lifespan of 84000km and a standard deviation of 13000km. What is the (approximate) probability that the average life for a random sample of 51 spark plugs will be between 80000km and 85000km?

4. Using the data in #16, p. 289, answer the following (**create the plots using Minitab, and pass in the plots**):

   (a) Construct a histogram of the data. Based on this plot, does it appear the data come from a normal distribution? Explain.

   (b) Construct a QQ-plot of the data. Based on this plot, does it appear the data come from a normal distribution? Explain.

5. A study of 400 kindergarten students showed that they had seen on average 5000 hours of television. If the sample standard deviation is 1000, find the 94% confidence interval of the mean hours of TV for all kindergarten students. If a parent claimed that his children watched 4000 hours, would the claim be believable?

6. #25, p. 307, **but change "within \$150" to "within \$180"**.

7. #12, p. 312.

8. #17, p. 312 **using Minitab**. Repeat this problem was repeated using a 98% confidence interval. Which interval (98% or 95%) is narrower? **Pass in the relevant Minitab output**.

9. #12, p. 319. **BUT, OMIT THE PART OF THE QUESTION STARTING WITH** *If the cafeteria manager ...*.

10. An Oct. 2009 national poll found that, in a random sample of 3320 Canadian voters, 890 favoured the federal Liberal party. Find a 99% confidence interval for the proportion of all Canadian voters that favour the federal Liberal party.

11. The company that makes *Tide* feels that university students do as little laundry as possible (imagine that!). They want to construct a confidence interval for the fraction of college students who do laundry once a week.

    (a) How large a sample do they need to use if they want to construct a 90% confidence interval that is accurate within 3%?

    (b) How large a sample do they need to use if they want to construct a 96% confidence interval that is accurate within 3%? How does this result compare to (a)?

    (c) How large a sample do they need to use if they want to construct a 90% confidence interval that is accurate within 2%? How does this result compare to (a)?

12. #13, p. 346. **NOTE: In (a)–(c), (f) and (g), take "is" to be "is not"**.

13. #4, p. 355. Also find the p-value.

STATS 2500-001

50 Points

SOLUTIONS : ASSIGNMENT #3

1 a Let X = number that own an iPod.

We know from Assignment #2 that X is binomial.
Here, $n = 277$, $p = 0.64$

We need $P(160 < X < 180)$.

Minitab gives cumulative probability : $P(X \leq \text{number})$.
So we write

$$P(160 < X < 180) = P(X \leq 179) - P(X \leq 160)$$

From Minitab output: $P(X \leq 160) =$ ~~[illegible]~~ 0.0186 1

$P(X \leq 179) = 0.6073$ 1

Then

$$P(160 < X < 180) = .6073 - 0.0186 = \boxed{0.5887}$$ 1

b We need $P(X \geq 156)$. By Complement rule:

$$P(X \geq 156) = 1 - P(X < 156) = 1 - P(X \leq 155).$$ 1

From Minitab output: $P(X \leq 155) = 0.0035$ 1

Then: $P(X \geq 156) = 1 - .0035 = \boxed{0.9965}$ 1

T

2

#1

Cumulative Distribution Function

Binomial with n = 277 and p = 0.64

```
  x  P( X <= x )
179     0.607344
```

$P(X \geq 179)$

a

Cumulative Distribution Function

Binomial with n = 277 and p = 0.64

```
  x  P( X <= x )
160    0.0186164
```

$P(X \geq 160)$

Cumulative Distribution Function

Binomial with n = 277 and p = 0.64

```
  x  P( X <= x )
155    0.0035183
```

b $P(X \geq 155)$

#2 (a) How much bread to produce?

If X = demand, $X \sim N(6800, \sqrt{84000}) = N(6800, 289.83)$

If we want to supply demand on 95% of days, we need to make an amount of bread X_0 such that the probability that the demand is this amount, or smaller, is 0.95!

$$P(X < X_0) = 0.95 \quad 1$$

Step 1: $P(Z < Z_0) = 0.95$

find 0.95 (or closest to it) in middle of normal table:

$\boxed{Z_0 = 1.645}$ or 1.64 or 1.65 1

Step 2: $Z = \frac{X-\mu}{\sigma}$

$$1.645 = \frac{x_0 - 6800}{289.83}$$

$x_0 = 289.83(1.645) + 6800 = \boxed{7276.77}$

which can be rounded to 7277 1

(don't take points off if not rounded)

3

4/

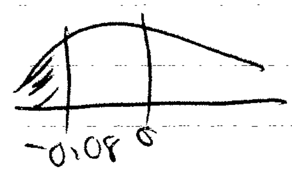
(2)

(b) If we are left with more than 500 loaves we sold (or demand) was less than $(7277-500)=6777$.

We need:

$$P(X<6777) = P\left(Z<\frac{6777-6800}{289.83}\right)$$ (1)

$$=P(Z<-0.08)$$ (1)



$$=\boxed{0.4681}$$ — (1)

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(3) "Probability that the average life..." use

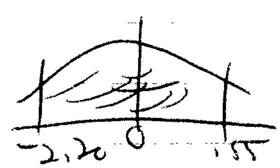
Central Limit Theorem: $\bar{X} \sim N(\mu, \sigma/\sqrt{n})$, $n\geq 30$.

$X=$ spark plug lifespan. $\mu=84000$, $\sigma=13000$, $n=51$.

$$P(80000<\bar{X}<85000)$$

$$=P\left(\frac{80000-84000}{13000/\sqrt{51}}<Z<\frac{85000-84000}{13000/\sqrt{51}}\right)$$

$$=P(-2.20<Z<0.55)$$ (1) (1)



$$=0.7088-0.0139=\boxed{0.6949}$$ (1)

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(4) (a) Histogram: Does not appear bell-shaped. (1)
Does not appear data are normally distributed.
(1)

(b) QQ-plot: Not linear (1) (lots of movement of points away from line). Does not appear data are normally distributed. (1)

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(5) CI for <u>mean</u> (μ) hours, n large: $\bar{X} \pm Z_{\alpha/2} \frac{S}{\sqrt{n}}$.

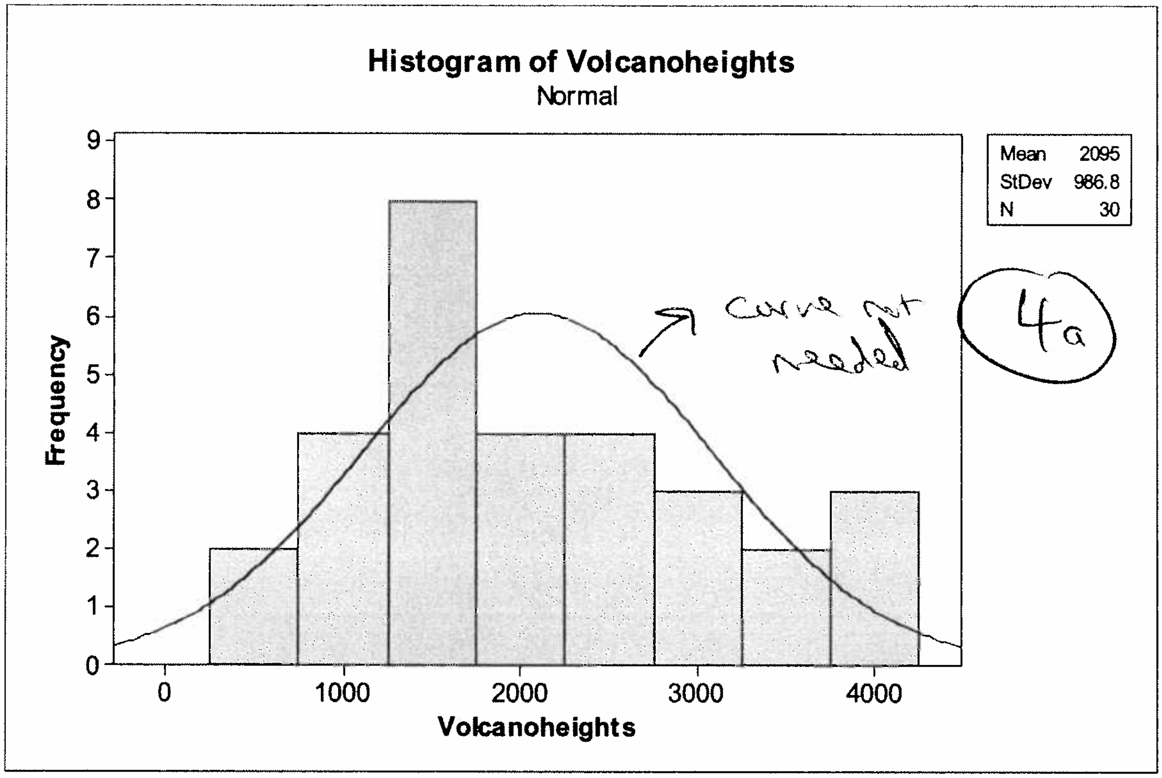
$\bar{X} = 5000$, $n = 400$, $S = 1000$.

(1)
94% CI: $\alpha = .06$, $\alpha/2 = .03$, $\boxed{Z_{.03} = 1.88}$ or 1.89

94% CI: $5000 \pm 1.88\left(\frac{1000}{\sqrt{400}}\right)$

$= 5000 \pm 94 = \boxed{(4906, 5094)}$ (1)

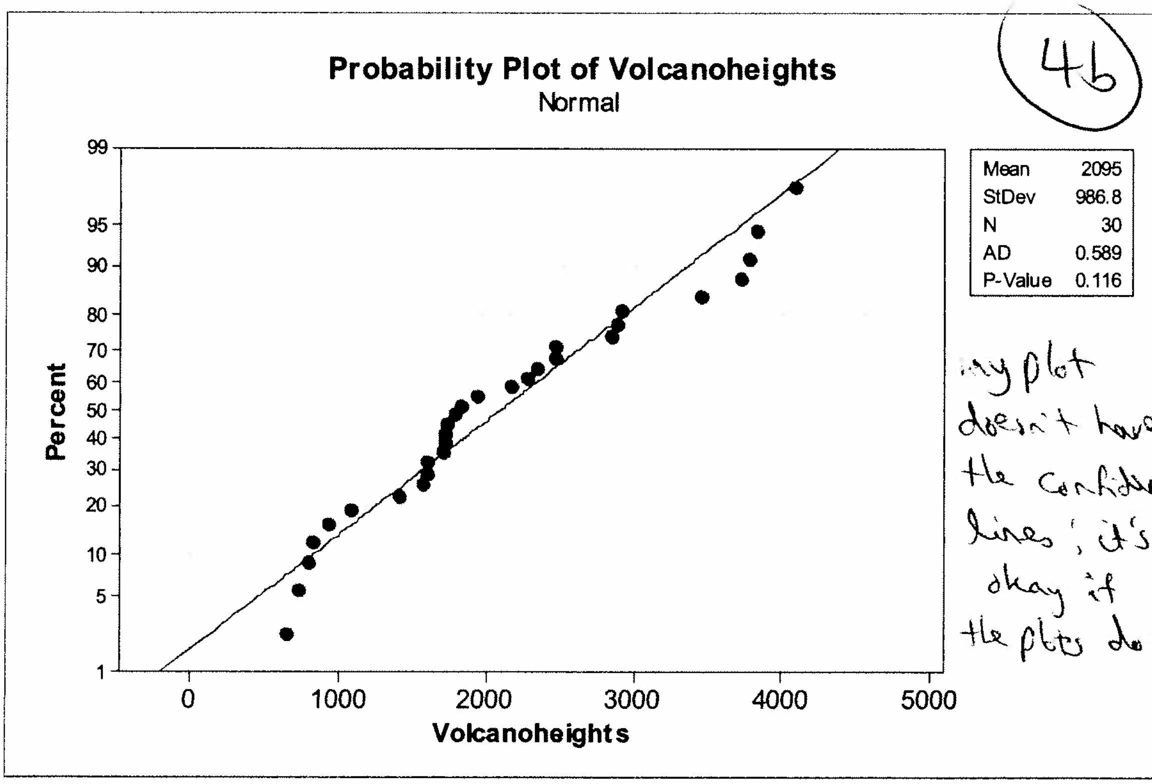
A claim that a child watched 4000 hours would not be believable, since 4000 falls outside of the CI for μ we obtained. (1)



curve not needed

4a

Probability Plot of Volcanoheights



4b

my plot doesn't have the confidence lines; it's okay if the plots do.

(6) ~~#~~25, p.307, change 150 to 180.

$\sigma = 1100$, $E = 180$ ("within 180").

90% CI $\Rightarrow$ $\alpha = .1$, $z_{.05} = 1.645$ or 1.64 or 1.65

$$n = \left[\frac{z_{\alpha/2} \cdot \sigma}{E}\right]^2 = \left[\frac{(1.645)(1100)}{180}\right]^2 = \boxed{101.06} \quad 1$$

So $\boxed{n = 102}$ 1

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(7) #12, p.312.

$n = 13$, $\bar{x} = 24$, $s = 2.7$.

99% CI, $n < 30$: $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$.

1

99% CI: $\alpha = .01$, $df = n-1 = 12$; $t_{.01/2} = \boxed{3.055}$

99% CI for μ: $24 \pm 3.055 \left(\frac{2.7}{\sqrt{13}}\right)$ 1

$= 24 \pm 2.288 = \boxed{(21.712, 26.288)}$

If he wants to use highest speeds to predict travel time of storms, use largest value in CI: 26.288 km/h. 1

(8) #17, p.312. Minitab output attached.

95% CI for μ: (38.79, 44.41) (1)
98% CI : (38.20, 45.00) (1)

The <u>95%</u> CI is <u>narrower</u>. (1)

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(9) #12, p.319

CI for <u>proportion</u> of students: $\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$\hat{p} = \frac{X}{n} =$ 0.28 (not given X, but given 28% directly)

$n = 350$.

<u>95% CI</u>: $\boxed{z_{.025} = 1.96}$

95% CI for p: $0.28 \pm 1.96\sqrt{\frac{(.28)(1-.28)}{350}}$

$= 0.28 \pm 0.047$

$= \boxed{(0.233, 0.327)}$ (2)

One-Sample T: carbs

Variable	N	Mean	StDev	SE Mean	95% CI
carbs	20	41.60	5.99	1.34	(38.79, 44.41)

#8

One-Sample T: carbs

Variable	N	Mean	StDev	SE Mean	98% CI
carbs	20	41.60	5.99	1.34	(38.20, 45.00)

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10) 99% CI for p.

$\hat{p} = \frac{X}{n} = \frac{890}{3320} = \boxed{0.268}$ ①

99% CI: $\alpha = .01$, $Z_{.005} = 2.575$ (or 2.57 or 2.58)

99% CI for p: $0.268 \pm 2.575\sqrt{\frac{.268(1-.268)}{3320}}$

$= 0.268 \pm 0.020$

$= \boxed{(0.248, 0.288)}$ ①

/2

11/

(11) (a) $n = \left(\frac{z_{\alpha/2}}{E}\right)^2 \hat{p}(1-\hat{p})$. ("fraction" = "proportion")

$\boxed{\hat{p} = 0.5}$ since unspecified (1)

$E = 0.03$ (within 3%)

90% CI: $z_{.05} = 1.645$ or 1.64 or 1.65

$n = \left(\frac{1.645}{.03}\right)^2 (.5)(.5) = \boxed{751.67}$ so $\boxed{n = 752}$
(1) (1)

(b) Same as (a), but now:

98% CI: $\alpha = .04$, $\frac{\alpha}{2} = .02$, $\boxed{z_{.02} = 2.05}$ or 2.06

$n = \left(\frac{2.05}{.03}\right)^2 (.5)(.5) = 1167.36$ so $\boxed{n = 1168}$
(1)

Higher CI $\Rightarrow$ need larger sample size [(b) is larger] (1)

(c) see (a), but $E = 0.02$

(1)

$n = \left(\frac{1.645}{.02}\right)^2 (.5)(.5) = 1691.27$ so $\boxed{n = 1692}$

(c) is larger (smaller margin of error needs a larger sample)
(1)

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(12) #13, p.346

(a) $H_0: \mu = 24.6$ (0.5)
$H_1: \mu \neq 24.6$

(b) $H_0: \mu = 51497$ (0.5)
$H_1: \mu \neq 51497$

(c) $H_0: \mu = 25.4$ (1)
$H_1: \mu < 25.4$

(d) $H_0: \mu = 88$ (0.5)
$H_1: \mu < 88$

(e) $H_0: \mu = 70$ (0.5)
$H_1: \mu < 70$

(f) $H_0: \mu = 79.95$ (0.5)
$H_1: \mu \neq 79.95$

(g) $H_0: \mu = 3.7$ (0.5)
$H_1: \mu \neq 3.7$

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(13) #4, p.355

$H_0: \mu = 48927$
$H_1: \mu > 48927$ (1)

$n = 44, \quad \bar{x} = 51607, \quad s = 2500$ (1)

Test stat: $z = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{51607 - 48927}{2500/\sqrt{44}} = \boxed{7.11}$

$\alpha = .05$: Reject H_0 if $z > z_{.05} = 1.96$. (1)
Since $7.11 > 1.96$, reject H_0 (accept H_1).
Lecturers do get higher salary. (1)

p-value $= P(z > 7.11) \approx 1 - .9999 = \boxed{0.0001}$ (1)

NOTE: Acceptable to have used p-value < α rule, and omitted rejection region method.

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