Tufts University
Department of Mathematics
Math 87 Homework 8

Due: Thursday, December 5, at 10:30 a.m. (in class).

1. (20 points) Solve the recurrence relation $a_{k+2} = 7a_{k+1} - 10a_k$, $a_0 = 2$, $a_1 = 7$.

2. (40 points) Consider a credit card that starts with a \$2000 balance and for which, each month, \$500 in new charges are accrued, and interest is collected on any unpaid balance at a rate of 1.5% per month (19.56% APR).

   (a) Write a recurrence relation for the balance after k months, b_k, when a fixed payment of p dollars is made each month.

   (b) Solve this recurrence relation for b_k as a function of p.

   (c) For what value of p does the monthly balance stay at \$2000?

   (d) Find an expression for the month, k, in which the balance becomes zero as a function of p.

   (e) How many months does it take to pay off the balance if $p =$ \$750? What about $p =$ \$1000?

   (f) How much should be paid each month to have a zero balance after exactly 6 months? After exactly 12 months? Use the bisection code from HW2 to compute the payments.

3. (40 points) For a given population of whales, an estimate of the annual growth rate of the population without harvesting is given by $rx(1-x/K)$, where $r = 0.06$ is the intrinsic growth rate, $K = 300,000$ is the maximum sustainable population, and x is the current population. (Notice that if $x = 0$ or $x = K$, the annual growth rate is zero, if $0 < x < K$, the annual growth rate is positive, while if $x > K$, the annual growth rate is negative.) If E boat-days of whaling are allowed per year, the annual growth rate is lowered by the amount $0.00001Ex$ (meaning that, for a population of x whales, each boat harvests 0.001% of the population for each day that it is whaling). Let the initial whale population be $x_0 = 70000$.

   (a) What is the steady-state population of whales when $E = 3000$? (The *steady-state* population occurs when the net growth rate is equal to zero.) What is the annual harvest when $E = 3000$ and the population is at its steady-state size?

   (b) Write a matlab code to compute the population after k years, for given k when $E = 3000$.

   (c) How many years does it take for the whale population to pass two-thirds of its steady-state size? Five-sixths? Eleven-twelfths? Note: you can also use the bisection code from HW2 to answer these questions, but must be careful to compute populations in integer numbers of years. The `floor` function can be helpful to do this. You could also find these points by graphing the population from year-to-year.

   (d) Noting that the growth in the population is quite slow, the whalers decide to refrain from whaling for a number of years to allow the population to grow quickly for a while, leading to greater harvests sooner. If there is no whaling ($E = 0$), how many years does it take for the whale population to pass two-thirds of its steady-state size? Five-sixths? Eleven-twelfths?

(e) Not whaling for enough time for the population to grow close to its steady-state value is also highly unprofitable for the whalers. Design a time-dependent strategy for picking E_k, the number of boat-days of whaling in year k (as a function of x_k or x_{k-1}), that also allows fast growth of the population to a its steady-state, as well as a non-zero harvest rate for the early years. Compare how long it takes the whale population to pass two-thirds, five-sixths, and eleven-twelfths of its steady-state size with your answers from the previous two parts.