

Tufts University
Department of Mathematics
Math 87 Homework 1

Due: Thursday, September 19, at 10:30 a.m. (in class).

1. A car manufacturer makes a profit of \$2,000 per car that it sells. In the past year, it sold 100,000 cars. In order to increase sales next year, it plans to offer rebates, estimating a linear relationship where sales will increase by 10% for each \$100 of rebate that it offers.
 - (a) (10 points) Plot the profit vs. rebate function (assuming the linear relationship holds for all rebate amounts).
 - (b) (10 points) What value of rebate will maximize its annual profit within this model?
 - (c) (5 points) What value of rebate would maximize its annual profit within this model if the company had sold 200,000 cars in the past year?
 - (d) (15 points) How sensitive is the optimal rebate value to the assumption that sales increase by exactly 10% for each \$100 of rebate? Explain why your answer makes sense. Plot the optimal value of the rebate as a function of the percentage increase in sales.
2. (25 points) The Cobb-Douglas production function is a model for the value of goods produced in an economic system, P in terms of values of labor, L , and capital, K , used in production. A generic Cobb-Douglas production function takes the form

$$P = CL^\beta K^\alpha,$$

where β and α are the output elasticities of labor and capital, representing the change in output expected from a change in input. A typical economy is constrained, in that $pL + qK = B$, where p and q are the costs of labor and capital in dollars per unit and B is the total available budget. Find the value of L that optimizes P in terms of the other variables. Find the sensitivity of this value of β .

3. Human blood is generally classified in the “ABO” system, with four blood types: A, B, O, and AB. These four types reflect six gene pairs (genotypes), with blood type A corresponding to gene pairs AA and AO, blood type B corresponding to gene pairs BB and BO, blood type O corresponding to gene pair OO, and blood type AB corresponding to gene pair AB. Let p be the proportion of gene A in the population, q be the proportion of gene B in the population, and r be the proportion of gene O in the population. Note that $p + q + r = 1$.
 - (a) (10 points) The Hardy-Weinberg principle states that p , q , and r are fixed from generation to generation, as are the frequencies of the different genotypes. Under this assumption, what is the probability that an individual has genotype AA? BB? OO? What is the probability of an individual having two different genes?
 - (b) (25 points) Find the maximum percentage of the population that can have two different genes under the Hardy-Weinberg principle in two different ways, by directly minimizing a function of only two variables and by using the method of Lagrange multipliers.