Tufts University
Department of Mathematics
Math 87 Homework 4

Due: Thursday, October 25, at 1:30 p.m. (in class).

1. (40 points) Consider the primal linear program

$$\begin{aligned} \text{Maximize } & 11x_1 + 5x_2 \\ \text{subject to } & x_1 + x_2 \leq 7 \\ & 10x_1 + 4x_2 \leq 40 \\ & x_1, x_2 \geq 0 \end{aligned}$$

   (a) Write the dual linear program.

   (b) Find the solutions to both the primal and the dual LPs by plotting the feasible sets. Confirm that both the strong duality theorem and complementary slackness are satisfied. What are the dual prices of each of the constraints?

   (c) Does the dual price provide an accurate prediction of the increase in the primal objective function when the right-hand side of the first constraint is increased by 1? By 2? By 4?

2. (60 points) Consider the linear program

$$\begin{aligned} \text{Maximize } & 50x_1 + 6x_2 + 35x_3 + 60x_4 \\ \text{subject to } & 24x_1 + 76x_2 + 43x_3 + 754x_4 \leq 800 \\ & 755x_1 + 27x_2 + 33x_3 + 67x_4 \leq 850 \end{aligned}$$

   (a) Restrict x_1, x_2, x_3, and x_4 to be integer variables that can take only the values 0 and 1. Use the branch and bound algorithm to find the optimal solution, explaining your choices for which variables to branch on and where to prune the tree. Draw the branch and bound tree for your solution. Note that you should use linprog in matlab to solve the relaxed LPs, initially with constraints that $0 \leq x_i \leq 1$ for $i = 1, 2, 3, 4$.

   (b) Repeat part (a) now allowing x_i to take values 0, 1, and 2, for $i = 1, 2, 3, 4$.