**Homework 6**

(Due Wednesday, May 12, 2010 -**NO late assignments will be accepted**)

1. Let $A$ be the finite-element matrix that you found for the one-dimensional diffusion equation in HW 3, discretized on a uniform mesh with $n = 2^\ell$ unknowns (not counting the left endpoint). That is, $A$ corresponds to the linear finite-element discretization of $-u''(x) = f(x)$ on $(0, 1)$ with $u(0) = 0$ and $u'(1) = 0$. Implement a two-grid scheme using linear interpolation and its transpose as restriction. Use your Gauss-Seidel code from HW5 as the relaxation procedure.
2. Let $A$ be the finite-element matrix that you found for the one-dimensional diffusion equation in HW 3, discretized on a uniform mesh with $n = 2^\ell$ unknowns (not counting the left endpoint). That is, $A$ corresponds to the linear finite-element discretization of $-u''(x) = f(x)$ on $(0, 1)$ with $u(0) = 0$ and $u'(1) = 0$. Implement a multigrid scheme using linear interpolation and its transpose as restriction. Use your Gauss-Seidel code from HW5 as the relaxation procedure.
3. Test the convergence of each of your methods by using them to solve $Ax = 0$ with a random initial guess for $x$. Plot the norms of the residual and error as a function of iteration for one realization of the initial guess. Also plot the relative reduction in residual norm per iteration, $\frac{\|0-Ax^{(\ell)}\|}{\|0-Ax^{\ell-1}\|}$. What can you say about the "asymptotic" convergence as $\ell$ gets large? Does this depend on $n$?