Homework 5

(Due Friday, April 9, 2010)

1. Let A be the finite-element matrix that you found for the one-dimensional diffusion equation in HW 3, discretized on a uniform mesh with n unknowns. That is, A corresponds to the linear finite-element discretization of $-u''(x) = f(x)$ on $(0,1)$ with $u(0) = 0$ and $u'(1) = 0$. Implement the Jacobi and Gauss-Seidel iterations to solve $Ax = b$. *Note:* There are several possible ways to do this, including to have generic Jacobi and Gauss-Seidel functions that take A as an input, or to have functions that are specific to this choice of A for a given value of n. You are free to implement this in any way that you choose.
2. Test the convergence of your method by using it to solve $Ax = 0$ with a random initial guess for x. (Why is this a good test problem?) Plot the norms of the residual and error as a function of iteration for one realization of the initial guess. Also plot the relative reduction in residual norm per iteration, $\frac{\|0-Ax^{(\ell)}\|}{\|0-Ax^{\ell-1}\|}$. What can you say about the "asymptotic" convergence as ℓ gets large? Does this depend on n?
3. Test the convergence of your method with initial guesses $x_i^{(0)} = \sin((k+\frac{1}{2})\pi\frac{i}{n})$. What can you say about how convergence depends on k? Does this depend on n?