MATH 2260 (Ordinary Differential Equations I) — Winter 2015
Practice Midterm Exam #2

1. (10 points) Consider the equation $x^2y'' - 6xy' + 12y = 0$.

   (a) Give an interval on which the equation (with suitable initial conditions) is guaranteed to have a unique solution.

   (b) Show that $y_1(x) = x^3$ and $y_2(x) = x^4$ are solutions of the equation on that interval.

   (c) Use the Wronskian to show that $\{y_1, y_2\}$ form a fundamental set of solutions on that interval.

2. (20 points) Find the general solution of each of the following equations.

   (a) $y'' - 5y' - 14y = 0$

   (b) $y'' - 2y' + 2y = 0$

   (c) $y'' - 2y' - y = 0$

   (d) $9y'' - 6y' + y = 0$

3. (20 points) Solve the initial value problem $4y'' - 4y' + 101y = 0$, $y(0) = -4$, $y'(0) = 13$.

4. (5 points) Determine a second-order linear homogeneous equation with constant coefficients for which $y(x) = 7xe^{-4x}$ is a solution.

5. (10 points) Find the general solution of $y'' + 4y = e^x + 1$.

6. (15 points) Find the general solution of $y'' - 2y' + y = e^x \ln x$ for $x > 0$.

7. (20 points) One solution of the equation $4x^2y'' + 8xy' + y = 0$ for $x > 0$ is $y_1(x) = \frac{1}{\sqrt{x}}$. Use the method of reduction of order to find a distinct second solution, $y_2(x)$, to the equation. Use the Wronskian to prove that $\{y_1, y_2\}$ is a fundamental set of solutions to the equation.