MATH 2260 (Ordinary Differential Equations I) — Winter 2015
Practice Final Exam

1. (5 points) Compute $y(2)$, where $y(x)$ is the solution of $xy' = 2y$ with $y(1) = 3$.

2. (5 points) At time $t = 0$, there are 5 grams of a radioactive substance present in a sample. At time $t = 2$, there are 4 grams present. Find the mass of the substance present as a function of time, t.

3. (10 points) Find the general solution of $xy^3y' = y^4 + x^4$ for $x > 0$.

4. (10 points) Consider the equation

$$(3x + 2y)dx + (x^2 + x + xy)dy = 0.$$

   (a) Show that the equation is not exact.

   (b) Show that $\mu(x, y) = xe^y$ is an integrating factor for this equation.

   (c) Find an implicit solution of the equation.

5. (5 points) Are the functions xe^x, $(x + 1)e^x$ and xe^{x+1} linearly independent for all x?

6. (20 points) Find the general solutions to the following ODEs

   (a) $(D^2 + 6D + 58)y = 0$

   (b) $(D - 4)^2(D^2 - 4)(D^2 + 4)y = 0$

   (c) $(D^2 + 4D + 20)(D^2 + 10D + 26)y = 0$

   (d) $(D - 3)^3y = e^{4x}$

7. (10 points) Find the general solution of $y'' + y = \sec x$ for $-\pi/2 < x < \pi/2$.

8. (10 points) Given that $\{1, x, x^2, 1/x\}$ is a fundamental set of solutions to $xy^4+4y''' = 0$, find the general solution to $xy^{(4)} + 4y''' = 6/x^2$.

9. (5 points) Using the definition of the Laplace Transform, compute $\mathcal{L}[e^{7t}]$. No credit will be given for an answer that does not use the definition.

10. (10 points) Use Laplace Transforms to solve

$$\begin{aligned} &(D^2 + 3D + 2)x = 10\sin(2t), \\ &x(0) = x'(0) = 0. \end{aligned}$$

   No credit will be given for a solution using any other technique.

11. (10 points) Solve $D^3x - Dx = \begin{cases} 1 & \text{if } t < 2 \\ 0 & \text{if } t \geq 2 \end{cases}$, $x(0) = x'(0) = x''(0) = 0$