

**MATH 2260 (Ordinary Differential Equations I) — Winter 2015**  
**Homework #7**

**Due Date:** Wednesday, March 25th, in class or in marking box #31 by 5:00 PM. You must show all work to receive credit.

1. (10 points) Check that  $y_1(x) = 1$ ,  $y_2(x) = e^{2x}$ , and  $y_3(x) = e^{-2x}$  are solutions of the equation  $y''' - 4y' = 0$  for all  $x$ . Is  $\{y_1, y_2, y_3\}$  a fundamental set?
2. (5 points each) Find the general solution for each of the following ODEs.
  - (a)  $y''' - 3y'' + 3y' - y = 0$
  - (b)  $y''' - y'' + 16y' - 16y = 0$
  - (c)  $y''' + 5y'' + 9y' + 5y = 0$
  - (d)  $y^{(4)} + 12y'' + 36y = 0$
  - (e)  $(D^2 + 9)^3 D^2 y = 0$
  - (f)  $(D^2 + 1)(D^2 + 9)^2(D - 2)y = 0$
3. (10 points) Solve the initial value problem,  $y^{(4)} - 16y = 0$ ,  $y(0) = 2$ ,  $y'(0) = -2$ ,  $y''(0) = 0$ ,  $y'''(0) = 24$ .
4. (5 points each) Give the form of the particular solution to the following equations. Do not evaluate the coefficients in the guess.
  - (a)  $(D^2 + 4)^2(D^2 - 4)y = \cos(x) + \sin(2x) + e^{4x}$
  - (b)  $(D^2 + 2D + 2)(D^2 + 2)(D + 2)^2 = e^{2x} + x^2 e^{-2x} + \cos(2x)$
5. (10 points each) Find particular solutions for each of the following ODEs.
  - (a)  $y''' + 3y'' - y' - 3y = e^x(-2 + 4x + 24x^2)$
  - (b)  $y^{(4)} - 2y'' + y = -e^{-x}(8 - 48x)$
  - (c)  $xy^{(4)} + 4y''' = 6 \ln|x|$ . For  $x > 0$ , the general solution of  $xy^{(4)} + 4y''' = 0$  is  $c_1 + c_2x + c_3x^2 + c_4/x$ .
  - (d)  $L[y] = x(x^2 - 6)y^{(4)} + 2(x^2 - 12)y''' + x(6 - x^2)y'' + 2(12 - x^2)y' = 2(x^2 - 6)^2$ . For  $0 < x < \sqrt{6}$ , the general solution of  $L[y] = 0$  is  $c_1 + c_2/x + c_3e^x + c_4e^{-x}$ .