MATH 2260 (Ordinary Differential Equations I) — Winter 2015
Homework #5

Due Date: Wednesday, March 4th, in class or in marking box #31 by 5:00 PM. You must show all work to receive credit.

1. (5 points each) Find the general solution of the following equations

   (a) $y'' + 2y' + 10y = 0$

   (b) $y'' - 8y' + 16y = 0$

   (c) $4y'' + 4y' + 10y = 0$

   (d) $y'' + 14y' + 50y = 0$

2. (10 points) Find the solution of $16y'' + y = 0$, $y(0) = 2$, $y'(0) = 9$.

3. (10 points) Find the solution of $4y'' - 12y' + 9y = 0$, $y(0) = 3$, $y'(0) = 5/2$.

4. (10 points) Given that $y_1(x) = e^{2x}$ is a solution of

$$xy'' - (4x + 1)y' + (4x + 2)y = 0,$$

   Find a second solution, $y_2(x)$, using reduction of order, and verify that $y(x) = c_1 y_1(x) + c_2 y_2(x)$ is the general solution of the ODE for $x > 0$.

5. (20 points) Find a solution of the form $y(x) = x^\alpha$ (for real-valued α) to the ODE $x^2 y'' - xy' + y = 0$ for $x > 0$. Use this solution with reduction of order to find the general solution of the ODE. You must prove that the two solutions you find are linearly independent and explicitly verify that they satisfy the ODE to receive full credit.

6. (10 points each) Find particular solutions for the following nonhomogeneous ODEs.

   (a) $y'' - 3y' + 2y = (1 + x)e^{3x}$

   (b) $y'' + 2y' + y = (2 + 3x)e^{-x}$

   (c) $y'' + 2y' + y = (6\cos(x) + 17\sin(x))\, e^x$