MATH 2260 (Ordinary Differential Equations I) — Fall 2014
Practice Final Exam #2

Instructions: No calculators, books, or notes are allowed on this exam. All electronic devices must be turned off and put away. Unless noted as "no partial credit", you must show all your work to receive any credit (full or partial).

1. (10 points) Find the general solution of

$$\cos(x)y' + \sin(x)y = \cos(x)\sin(x)$$

   for $-\pi/2 < x < \pi/2$.

2. (10 points) An oil tanker carrying 100,000 gallons of oil runs aground offshore at $t = 0$. Fresh water pours into one end of the tanker at a rate of 1000 gallons per hour, while the mixture of water and oil pours out the other end, also at the rate of 1000 gallons per hour. Solve for the amount of oil in the tanker at as function of t.

3. (10 points) Consider the equation

$$(3x + 2y)dx + (x^2 + x + xy)dy = 0.$$

   (a) Show that the equation is not exact.

   (b) Show that $\mu(x, y) = xe^y$ is an integrating factor for this equation.

   (c) Find an implicit solution of the equation.

4. (5 points) Are the functions xe^x, $(x + 1)e^x$ and xe^{x+1} linearly independent for all x?

5. (15 points) Find the general solution of

   (a) $(D^2 - 9)^2(D^2 + 9)y = 0$

   (b) $(D^2 + 6D + 58)y = 0$

   (c) $(D - 3)^3y = e^{4x}$

6. (15 points) Consider the equation $(xD^3 + 3D^2)y = x^{-1/2}$ for $x > 0$.

   (a) Find three solutions of the form x^α to the related homogeneous equation.

   (b) Find the general solution to the original (nonhomogeneous) equation. You may assume that your solutions from part (a) form a fundamental set.

7. (10 points) Let α and β be distinct positive constants.

   (a) Find the solution of the undampled spring equation,

$$\begin{aligned}(D^2 + \alpha^2)x &= \cos(\beta t),\\ x(0) &= x'(0) = 0.\end{aligned}$$

(b) Use the identity $\cos(\theta_1 \pm \theta_2) = \cos(\theta_1)\cos(\theta_2) \mp \sin(\theta_1)\sin(\theta_2)$ to show that $x(t) = C \sin\left(\frac{(\alpha-\beta)t}{2}\right) \sin\left(\frac{(\alpha+\beta)t}{2}\right)$.

8. (5 points) Compute $\mathcal{L}^{-1}\left[\frac{3s}{(s-2)^5}\right]$.

9. (10 points) Use Laplace Transforms to solve the initial value problem

$$\begin{aligned}&(D^2 + 2D - 3)x = 3e^t,\\ &x(0) = 1, x'(0) = 0.\end{aligned}$$

No credit will be given for a solution using any other technique.

10. (10 points) Solve $D^2x - x = 3\delta(t-2)$, $x(0) = 2$, $x'(0) = 0$