MATH 2260 (Ordinary Differential Equations I) — Fall 2014
Practice Final Exam #1

Instructions: No calculators, books, or notes are allowed on this exam. All electronic devices must be turned off and put away. Unless noted as "no partial credit", you must show all your work to receive any credit (full or partial).

1. (5 points) Compute $y(2)$, where $y(x)$ is the solution of $xy' = 2y$ with $y(1) = 3$.

2. (5 points) At time $t = 0$, there are 5 grams of a radioactive substance present in a sample. At time $t = 2$, there are 4 grams present. Find the mass of the substance present as a function of time, t.

3. (10 points) Find the general solution of $xy^3y' = y^4 + x^4$ for $x > 0$.

4. (15 points) Find the general solutions to the following ODEs

   (a) $(D-4)^2(D^2-4)(D^2+4)y = 0$

   (b) $(D^2+4D+20)(D^2+10D+26)y = 0$

   (c) $(D^2+2D+1)y = 2x+3$

5. (15 points) Consider the equation $x^2y'' - xy' + y = 0$ for $x > 0$.

   (a) Given that $y_1(x) = x\ln(x)$ is a solution of the equation (you do not need to check this), use the method of reduction of order to find a second solution.

   (b) Do these two solutions form a fundamental set of solutions to the ODE?

6. (10 points) Find the general solution of $y'' + y = \sec x$ for $-\pi/2 < x < \pi/2$.

7. (10 points) Given that $\{1, x, x^2, 1/x\}$ is a fundamental set of solutions to $xy^4 + 4y''' = 0$, find the general solution to $xy^{(4)} + 4y''' = 6/x^2$.

8. (5 points) Consider a spring system with spring constant $k = 4$ kg/s^2 and mass $m = 1$ kg. Give damping factors b (in units of kg/s) for which the system is

   (a) underdamped,

   (b) overdamped,

   (c) critically damped.

9. (5 points) Using the definition of the Laplace Transform, compute $\mathcal{L}[e^{7t}]$. No credit will be given for an answer that does not use the definition.

10. (10 points) Use Laplace Transforms to solve

$$(D^2+3D+2)x = 10\sin(2t),$$
$$x(0) = x'(0) = 0.$$

    No credit will be given for a solution using any other technique.

11. (10 points) Solve $D^3x - Dx = \begin{cases} 1 & \text{if } t < 2 \\ 0 & \text{if } t \geq 2 \end{cases}$, $x(0) = x'(0) = x''(0) = 0$