

1. (20 points) Let $\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n$ be vectors in \mathbb{R}^m . Define each of the following terms

- (a) $\text{span}\{\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n\}$
- (b) Vectors \vec{u}_1 and \vec{u}_2 that are *parallel*
- (c) The standard basis vectors \vec{i}, \vec{j} , and \vec{k} in \mathbb{R}^3
- (d) $\vec{v} \times \vec{w}$ for vectors \vec{v}, \vec{w} in \mathbb{R}^3

2. (25 points) Consider the vectors $\vec{u} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 3 \\ 0 \\ -2 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$.

- (a) Compute $\vec{u} \cdot (2\vec{v} + \vec{w})$.
- (b) Is \vec{w} in the plane spanned by \vec{u} and \vec{v} ?
- (c) Find the equation for the plane spanned by \vec{v} and \vec{w} .

3. (15 points) Consider the plane given by $3x + y - 2z = 1$ and the line given by

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

- (a) Find the point of intersection between the line and the plane.
- (b) Find the line through this point in the direction of the normal vector to the plane.

4. (20 points) Consider points $A(0, 1, 0)$, $B(2, 0, 1)$, and $C(1, -1, 0)$, and the parallelogram with sides \overrightarrow{AB} and \overrightarrow{AC} .

- (a) What are the coordinates of the fourth vertex of the parallelogram?
- (b) Find the cosine of the angle formed by the sides of the parallelogram at point B .
- (c) Find the area of the parallelogram.

5. (20 points) Let \vec{u} and \vec{v} be vectors in \mathbb{R}^n .

- (a) State the Cauchy-Schwarz inequality
- (b) Use the Cauchy-Schwarz inequality to prove the triangle inequality, that

$$\|\vec{u} + \vec{v}\| \leq \|\vec{u}\| + \|\vec{v}\|.$$