**MATH 2050** **Practice Midterm 2** **Fall 2015**

1. (20 points) Consider the plane $\pi : 2x - y + z = 0$.

   (a) Find two orthogonal vectors in the plane, $\pi$.

   (b) Find the projection of $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ onto the plane, $\pi$.

2. (15 points) Consider $\vec{u}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, $\vec{u}_3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, and $\vec{u}_4 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$.

   (a) Show that $\vec{u}_1$, $\vec{u}_2$, and $\vec{u}_3$ are linearly independent.

   (b) Are $\vec{u}_1$, $\vec{u}_2$, $\vec{u}_3$, and $\vec{u}_4$ linearly independent?

3. (10 points) Given $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, compute $AB$ and $BA$.

4. (40 points) Find all solutions, if any, to each of the given systems of equations. Express your answers in vector form.

   (a) $\begin{aligned} 2x - y &= 3 \\ 4x - 2y &= 4 \end{aligned}$

   (b) $\begin{aligned} x - y + 2z &= -2 \\ 2y - z &= -4 \\ x + y + 3z &= -10 \end{aligned}$

   (c) $\begin{aligned} x + y + 3z &= 2 \\ -2x - 2y - 6z &= -4 \end{aligned}$

   (d) $\begin{aligned} x + y - 2z &= 3 \\ 2x - y + 3z &= 0 \\ x - 2y + 2z &= -3 \end{aligned}$

5. (15 points) Let $\vec{u}$ and $\vec{v}$ be vectors in $\mathbb{R}^n$.

   (a) Give the formula for $\text{proj}_{\vec{v}}\vec{u}$.

   (b) Show that if $\vec{u}$ is nonzero, then $\vec{u} - \text{proj}_{\vec{u}}\vec{u}$ is the zero vector.