1. (20 points) Let $\vec{u}$ and $\vec{v}$ be vectors in $\mathbb{R}^n$.
   (a) State the *Cauchy-Schwarz Inequality.*
   (b) State the *Triangle Inequality.*
   (c) Give the definitions of $\|\vec{u}\|$ and $\vec{u} \cdot \vec{v}$.
   (d) Define the *angle* between $\vec{u}$ and $\vec{v}$.
   (e) Give the definition of when $\vec{u}$ and $\vec{v}$ are *parallel.*

2. (20 points) Consider the vectors $\vec{u} = \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix}$.
   (a) Is $\vec{w} = \begin{bmatrix} 2 \\ 6 \\ -4 \end{bmatrix}$ a linear combination of $\vec{u}$ and $\vec{v}$?
   (b) Find a vector orthogonal to both $\vec{u}$ and $\vec{v}$
   (c) Find the area of the parallelogram with sides $\vec{u}$ and $\vec{v}$.

3. (20 points) Find the point of intersection of the lines $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} + t \begin{bmatrix} 5 \\ 0 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$. Find the equation of the plane that includes both lines.

4. (10 points) Consider the points $A(0,1,2)$, $B(2,3,1)$, and $C(2,2,4)$.
   (a) Find the cosine of the angle between vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.
   (b) Are $\overrightarrow{AB}$ and $\overrightarrow{AC}$ orthogonal?

5. (15 points) Consider the vectors $\vec{u} = \begin{bmatrix} 0 \\ 3 \\ -2 \end{bmatrix}$, and $\vec{v} = \begin{bmatrix} 1 \\ x \\ 3x^2 \end{bmatrix}$.
   (a) Find all values of x such that $\vec{u}$ and $\vec{v}$ are orthogonal.
   (b) Explain why there are no values of x such that $\vec{u}$ and $\vec{v}$ are parallel.
   (c) Give a non-zero vector $\vec{w}$ that is orthogonal to $\vec{v}$ for all values of x.

6. (15 points) Verify the *scalar triple product* identity, that, for any vectors $\vec{u}$, $\vec{v}$, and $\vec{z}$ in $\mathbb{R}^3$,

$$\vec{u} \cdot (\vec{v} \times \vec{z}) = \vec{v} \cdot (\vec{z} \times \vec{u}) \,.$$