

MEMORIAL UNIVERSITY OF NEWFOUNDLAND  
DEPARTMENT OF MATHEMATICS AND STATISTICS

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MIDTERM TEST

**MATHEMATICS 1000**

OCTOBER 18TH, 2002

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**Name**

**MUN Number**

1. Evaluate each of the following limits, if it exists. Assign, if possible,  $+\infty$  or  $-\infty$  to any limit that does not exist.

[4] (a)  $\lim_{x \rightarrow -3} \frac{1 - \sqrt{10 + 3x}}{3 + x}$

[4] (b)  $\lim_{x \rightarrow -4} \frac{x^2 - 16}{2x^2 + 15x + 28}$

[4] (c)  $\lim_{t \rightarrow 0} \frac{\sin^2(3t)}{t^2}$

2. Consider the following function:

$$g(x) = \begin{cases} \frac{|x-2|}{x-2}, & x < 2 \\ -1, & x = 2 \\ \frac{-8(2-x)}{x^3 - 8x^2 + 12x}, & x > 2 \end{cases}$$

[8] (a) Find the vertical asymptotes of  $g(x)$ . Justify your answers using limits, and classify any discontinuities.

[6] (b) Is  $g(x)$  continuous at  $x = 2$ ? Use the definition of continuity to support your answer.

3. Let  $f(x) = \sqrt{5 - 2x}$ .

[7] (a) Use the definition of the derivative to find  $f'(x)$ .

[3] (b) Find the equation of the tangent line to  $f(x)$  at  $x = \frac{1}{2}$ .

4. Find the derivative of the following functions using the appropriate differentiation rules, making any obvious simplifications.

[4] (a)  $f(x) = \frac{x^3 + 2x - 1}{\sqrt{x}}$

[4] (b)  $g(t) = \frac{\cos(t)}{1 + \sin(t)}$

[6] (c)  $h(x) = (2 + \tan(e^{4x})) (x^2 + 7x)^2$

**Bonus Question:**

The mass of glucose in a metabolic experiment decreases according to the equation

$$m(t) = 50 - 2t^{\frac{3}{2}},$$

where  $m(t)$  = mass of glucose in grams (g) at time  $t$ , and  $t$  is time in hours.

- (a) Find the initial mass of glucose used in the experiment.
- (b) Find the rate of change in the amount of glucose  $\left(\frac{dm}{dt}\right)$  after 4 hours. How much glucose has been metabolized (i.e. used) after 4 hours?