SOLUTIONS

1. (a) $\int 10(5x+4)^8\,dx = 10\int (5x+4)^8\,dx = 10\left(\frac{1}{9}\right)\left(\frac{1}{5}\right)(5x+4)^9 + C = \frac{2}{9}(5x+4)^9 + C$

   (b) $\int e^{\left(6-\frac{x}{2}\right)}\,dx = \left(\frac{1}{-\frac{1}{2}}\right)e^{\left(6-\frac{x}{2}\right)} + C = -2e^{\left(6-\frac{x}{2}\right)} + C$

   (c) $\int \frac{t^5+3t^3+\sqrt{t}}{9t^4}\,dt = \int\left(\frac{t}{9}+\frac{1}{3t}+\frac{1}{9}t^{-\frac{7}{2}}\right)\,dt = \frac{t^2}{18}+\frac{1}{3}\ln|t| + \left(\frac{1}{9}\right)\left(-\frac{2}{5}\right)t^{-\frac{5}{2}} + C$
   $= \frac{t^2}{18}+\frac{1}{3}\ln|t| - \frac{2}{45}t^{-\frac{5}{2}} + C$

   (d) $\int \left[\sec^2(4x-1)+\sqrt{4x-1}\right]\,dx = \frac{1}{4}\tan(4x-1) + \left(\frac{1}{4}\right)\left(\frac{2}{3}\right)(4x-1)^{\frac{3}{2}} + C$
   $= \frac{1}{4}\tan(4x-1) + \frac{1}{6}(4x-1)^{\frac{3}{2}} + C$

   (e) $\int \csc\theta(\csc\theta-\cot\theta)\,d\theta = \int(\csc^2\theta - \csc\theta\cot\theta)\,d\theta = -\cot\theta + \csc\theta + C$

   (f) $\int 4\sin(-4x)\cos(-4x)\,dx = 2\int \sin(-8x)\,x = \frac{-2}{-8}\cos(-8x) + C = \frac{1}{4}\cos(-8x) + C$

   (g) $\int \frac{\left(1+\frac{1}{x}\right)\left(1+\frac{4}{x}\right)}{3x}\,dx = \int \frac{\left(1+\frac{5}{x}+\frac{4}{x^2}\right)}{3x}\,dx = \int\left(\frac{1}{3x}+\frac{5}{3}x^{-2}+\frac{4}{3}x^{-3}\right)\,dx$
   $= \frac{1}{3}\ln|x| - \frac{5}{3}x^{-1} + \left(\frac{4}{3}\right)\left(-\frac{1}{2}\right)x^{-2} + C = \frac{1}{3}\ln|x| - \frac{5}{3x} - \frac{2}{3x^2} + C$

   (h) $\int \frac{x^3+8}{x+2}\,dx = \int \frac{(x+2)(x^2-2x+4)}{x+2}\,dx = \int (x^2-2x+4)\,dx = \frac{1}{3}x^3 - x^2 + 4x + C$

2. We have

$$f'(x) = \int f''(x)\,dx = \int (x+2)\,dx = \frac{x^2}{2} + 2x + C$$

so since $f'(1) = \frac{1}{2} + 2 + C = 2$ then $C = -\frac{1}{2}$. Next,

$$f(x) = \int f'(x)\,dx = \int\left(\frac{x^2}{2} + 2x - \frac{1}{2}\right)\,dx = \frac{x^3}{6} + x^2 - \frac{1}{2}x + D$$

so since $f(0) = 0+0+0+D = 3$ then $D = 3$. Hence $f(x) = \frac{x^3}{6} + x^2 - \frac{1}{2}x + 3$.

3. We let $a(t) = -9.8$ so then

$$v(t) = \int a(t)\,dt = \int(-9.8)\,dt = -9.8t + C$$

and since $v(0) = 10$, $C = 10$. Next,

$$s(t) = \int v(t)\,dt = \int(-9.8t+10)\,dt = -4.9t^2 + 10t + D$$

and since $s(0) = 0$, $D = 0$. Let the time the ball reaches its maximum height be T, then $v(T) = 0$ and so $-9.8T + 10 = 0$ and $T = \frac{50}{49}$. Therefore, at this time,

$$s(T) = s\left(\frac{50}{49}\right) = -4.9\left(\frac{50}{49}\right)^2 + 10\frac{50}{49} = \frac{250}{49} \approx 5.10.$$

So the ball reaches a maximum height of about 5.10 metres.

4. Note that $F'(x) = e^{\cos(x)} - x^3$ so $F'(0) = e^1 - 0 = e$.

5. $g(x) = \frac{d}{dx}[e^{\cos(x)} - x^3] = -\sin(x)e^{\cos(x)} - 3x^2$

6. (a) $\int_{\frac{\pi}{8}}^{\pi} \cos(2x)\,dx = \left[\frac{1}{2}\sin(2x)\right]_{\frac{\pi}{8}}^{\pi} = \frac{1}{2}\left[\sin(2\pi) - \sin\left(\frac{\pi}{4}\right)\right] = \frac{1}{2}\left[0 - \frac{\sqrt{2}}{2}\right] = -\frac{\sqrt{2}}{4}$

(b) $\int_{-2}^{0} \frac{3u+8}{3u+7}\,du = \int_{-2}^{0} \frac{(3u+7)+1}{3u+7}\,du = \int_{-2}^{0} \left(1 + \frac{1}{3u+7}\right)du = \left[u + \frac{1}{3}\ln|3u+7|\right]_{-2}^{0}$
$= \left[0 + \frac{1}{3}\ln(7)\right] - \left[-2 + \frac{1}{3}\ln(1)\right] = 2 + \frac{\ln(7)}{3}$

(c) $\int_{2}^{0} (4x+1)^{-\frac{5}{2}}\,dx = \left[\left(\frac{1}{4}\right)\left(-\frac{2}{3}\right)(4x+1)^{-\frac{3}{2}}\right]_{2}^{0} = -\frac{1}{6}\left[1^{-\frac{3}{2}} - 9^{-\frac{3}{2}}\right] = -\frac{1}{6}\left[1 - \frac{1}{27}\right] = -\frac{13}{81}$

(d) $\int_{1}^{e} (3x^{-3} + 5x^{-1} - 6x^2)\,dx = \left[-\frac{3}{2}x^{-2} + 5\ln|x| - 2x^3\right]_{1}^{e}$
$= \left[-\frac{3}{2}e^{-2} + 5 - 2e^3\right] - \left[-\frac{3}{2} + 0 - 2\right] = \frac{17}{2} - \frac{3}{2e^2} - 2e^3$

7. $A = \int_{\frac{11}{4}}^{\frac{35}{4}} \frac{2}{\sqrt{2x - \frac{3}{2}}}\,dx = 2\left[\left(\frac{1}{2}\right)(2)\left(2x - \frac{3}{2}\right)^{\frac{1}{2}}\right]_{\frac{11}{4}}^{\frac{35}{4}} = [4-2] = 2$

8. (a) $A = \int_{1}^{2} \left(2 - \frac{1}{x^2}\right)dx = \left[2x + \frac{1}{x}\right]_{1}^{2} = \left[4 + \frac{1}{2}\right] - [2+1] = \frac{3}{2}$

(b) We solve for points of intersection: $x^2 + 3x = x + 3 \Longrightarrow x^2 + 2x - 3 = 0 \Longrightarrow (x+3)(x-1) = 0$ so $x = 1$ and $x = -3$. Note that $x + 3 \geq x^2 + 3x$ on $[-3, 1]$, so then $A = \int_{-3}^{1} [(x+3) - (x^2+3x)]\,dx = \int_{-3}^{1} [-x^2 - 2x + 3]\,dx = \left[-\frac{1}{3}x^3 - x^2 + 3x\right]_{-3}^{1} = \left[-\frac{1}{3} - 1 + 3\right] - [9 - 9 - 9] = \frac{32}{3}$.

(c) We solve for points of intersection: $\frac{(x+1)^2}{2} = x^3 + 1 \Longrightarrow 2x^3 - x^2 - 2x + 1 = 0 \Longrightarrow x^2(2x-1) - (2x-1) = 0 \Longrightarrow (x^2-1)(2x-1) = 0 \Longrightarrow (x+1)(x-1)(2x-1) = 0$ so $x = 1$, $x = -1$ and $x = \frac{1}{2}$. On $[-1, \frac{1}{2}]$, $x^3+1 \geq \frac{(x+1)^2}{2}$ and on $[\frac{1}{2}, 1]$, $\frac{(x+1)^2}{2} \geq x^3+1$. Hence $A = \int_{-1}^{\frac{1}{2}} \left[(x^3+1) - \frac{(x+1)^2}{2}\right]dx + \int_{\frac{1}{2}}^{1} \left[\frac{(x+1)^2}{2} - (x^3+1)\right]dx = \frac{45}{64} + \frac{7}{192} = \frac{71}{96}$.

(d) We solve for the points of intersection: $x^4 + 1 = 2x^2 \Longrightarrow x^4 - 2x^2 + 1 = 0 \Longrightarrow (x^2-1)^2 = 0 \Longrightarrow x^2 = 1 \Longrightarrow x = \pm 1$. Note that $x^4 + 1 \geq 2x^2$ on $[-1, 1]$, so

$$A = \int_{-1}^{1} [(x^4+1) - 2x^2]\,dx = \left[\frac{x^5}{5} + x - \frac{2}{3}x^3\right]_{-1}^{1} = \left[\frac{1}{5} + 1 - \frac{2}{3}\right] - \left[-\frac{1}{5} - 1 + \frac{2}{3}\right] = \frac{16}{15}.$$

(e) $A = \int_{-\frac{\pi}{8}}^{\frac{\pi}{8}} [\sec^2(2x) - x^2]\,dx = \left[\frac{1}{2}\tan(2x) - \frac{1}{3}x^3\right]_{-\frac{\pi}{8}}^{\frac{\pi}{8}} = \left[\frac{1}{2} - \frac{\pi^3}{1536}\right] - \left[-\frac{1}{2} + \frac{\pi^3}{1536}\right]$
$= 1 - \frac{\pi^3}{768}$

(f) Since $|x| = x$ for $x \geq 0$ and $|x| = -x$ for $x \leq 0$, we must check for points of intersections using both forms. First, set $x^2 = x$ so $x(x-1) = 0$ and $x = 0$ or $x = 1$. Next, set $x^2 = -x$ so $x(x+1) = 0$ and $x = 0$ or $x = -1$. Since all the points of intersection appear on the appropriate intervals, these are all points of intersection between $y = |x|$ and $y = x^2$ (for example, if we had found that a point of intersection between $y = x$ and $y = x^2$ was $x = -3$, this would have been invalid because $y = |x|$ does not have the form $y = x$ for $x < 0$). Note that on $[-1, 0]$, $-x > x^2$ and on $[0, 1]$, $x > x^2$. So then $A = \int_{-1}^{0} [-x - x^2]\,dx + \int_{0}^{1} [x - x^2]\,dx = \left[-\frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{0}^{1} = 0 - \left[-\frac{1}{2} + \frac{1}{3}\right] + \left[\frac{1}{2} - \frac{1}{3}\right] - 0 = \frac{1}{3}$.