Related Rates

Chapter 2: §2.7

The following are some examples of related rates questions which will be solved during class. Note the underlined words, they usually indicate an important piece of information required for the question. Section 2.7 can be found on pages 173–181 in the 2ndedition, and on pages 175–183 of the 3rdedition. The examples below are different from the worked examples in the text. **Read the questions carefully!**

1. Air is being pumped into a spherical balloon such that its volume increases at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm? $\left[\text{Note: Volume of a sphere is given by } V = \frac{4}{3}\pi r^3\right]$

2. A ladder 10 feet long rests against a vertical wall.

   (a) If the bottom of the ladder slides away from the wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?

   (b) Find the rate at which the angle θ between the ladder and the ground is changing (in radians/second) at the same moment as in (a).

3. A water tank has the shape of an inverted circular cone (i.e. the tip is pointing down and the base is a circle) with base radius 2 m and height 4 m. If water is being pumped into the tank at a constant rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3 m deep. $\left[\text{Note: Volume of a circular cone is given by } V = \frac{1}{3}\pi r^2 h\right]$

4. A trough is 10 ft long and and its ends have the shape of isosceles triangles that are 3 ft across at the top and have a height of 1 ft. If the trough is filled with water at a rate of 12 ft^3/min, how fast does the water level rise when the water is 6 inches deep? Remember that 6 inches = 0.5 ft.

   Note: Volume of a trough of this shape is given by $V = \frac{1}{2}bhl$, where b = base of the triangle, h = height, and l = length of the trough.

5. A plane flying with a constant speed of 300 km/h passes over a ground radar station at an altitude of 1 km and climbs at an angle of 30°. At what rate is the distance from the plane to the radar station increasing 1 min later? Note: Watch the dimensions.

6. A kite 100 ft above the ground moves horizontally at a speed of 8 ft/s. At what rate is the angle between the string and the horizontal decreasing when 200 ft of string have been let out?

7. Water is leaking out of an inverted conical tank at a constant rate of 10000 cm^3/min at the same time that water is being pumped into the tank at a constant rate. The tank has a height of 6 m and the circumference of the circular top is 8π m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank. Note that the circumference of a circle (C) is related to the radius (r) by: $C = 2\pi r$.

   $\left[\text{Note: Volume of a circular cone is given by} \quad V = \frac{1}{3}\pi r^2 h\right]$

8. A spherical snowball is melting in such a way that its volume is decreasing at a rate of 1 cm^3/min. At what rate is the diameter decreasing when the diameter is 10 cm?
   $\left[\text{Note: Volume of a sphere is given by} \quad V = \frac{4}{3}\pi r^3\right]$

9. A spotlight on the ground shines on the wall of a building 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is his shadow on the building decreasing when he is 4 m from the building?

Strategy for Solving Related Rates Problems

1. Read the problem **carefully**. Make sure you do not miss any needed information.

2. Draw a diagram is possible — this might help you to answer the question.

3. Introduce variables, e.g. call something x, something else y, etc. — quantities that are functions of time t.

4. Identify all given information and quantities to be determined (label them on the diagram if there is one).

5. Write an equation that relates the variables in the problem $\rightarrow$ use the diagram, similar triangles, Pythagoras, etc. If appropriate, try to eliminate one of the variables by substitution in terms of another variable.

6. Implicitly differentiate both sides of the equation with respect to time, t using the chain rule on any variables which are functions of t.

7. Substitute in the given information and solve for the unknown quantity.

Note: Do not substitute information into the equation before differentiating; this can only be done with **fixed lengths**, i.e. quantities which do not change with time (for example the length of a ladder, the dimensions of a trough, etc.). Also, the many formulae required for these questions **will not** be provided on the exam, so you must learn these. Finally, the only way to get good at these is to practice lots of problems!