

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

QUIZ #4

MATHEMATICS 1000

NOVEMBER 29TH, 2002

Name

MUN Number

1. Given

$$f(x) = \frac{2x}{x^2 + 1}, \quad f'(x) = \frac{2(1 - x^2)}{(x^2 + 1)^2}, \quad f''(x) = \frac{4x(x^2 - 3)}{(x^2 + 1)^3}$$

[7]

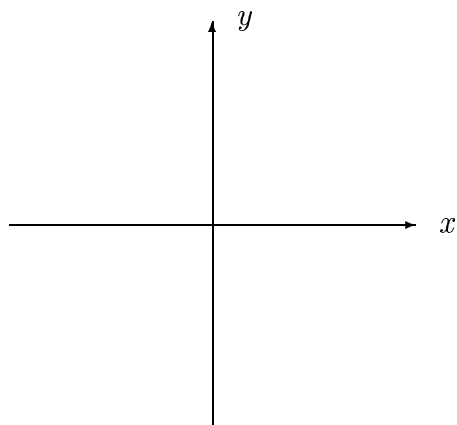
- (a) Find the critical numbers, and the intervals on which $f(x)$ is increasing, decreasing, concave up, and concave down. Find the exact coordinates of all extreme points, inflection points and intercepts. **You are not required to draw the graph.**

[3]

- (b) Find the equation(s) of any horizontal asymptotes of $f(x)$.

[8] 2. Sketch the graph of a function $y = f(x)$ with the following properties, on the axes provided.

- Domain of f : $\{x|x \neq -2\}$
- $f(0) = 0$, $f(1) = 1$, $f(2) = \frac{3}{2}$,
 $f(3) = 1$
- $f'(x) > 0$ for $0 < x < 2$
- $f'(x) < 0$ for $x < -2$, $-2 < x < 0$ and
 $x > 2$
- $f''(x) > 0$ for $-2 < x < 1$ and $x > 3$
- $f''(x) < 0$ for $x < -2$ and $1 < x < 3$
- $\lim_{x \rightarrow -2^-} f(x) = -\infty$, $\lim_{x \rightarrow -2^+} f(x) = +\infty$
- $\lim_{x \rightarrow -\infty} f(x) = \lim_{x \rightarrow \infty} f(x) = 0$



[7] 3. If 1200 cm^2 of material is available to make a box with a square base and an open top, find the largest volume of such a box.