

Worksheet on Differentiation and Related Rates

1. Find the derivatives of the following functions:

(a) $f(x) = \sqrt{\cos(x^2 e^{-x})}$

(b) $g(x) = \sin^2\left(x^{\frac{1}{3}} + \ln(x)\right)$

(c) $y = \frac{2x+1}{\sqrt{x^2+1}}$

(d) $h(x) = \ln|\cot(x) + \csc(x)|$

(e) $f(t) = 2^t \sec(2t)$

(f) $f(x) = \log_3(x^3 + 2x^2 - 1)$

(g) $y = \frac{(x^6+1)^3(x^4+1)}{14\sqrt{x+9}}$

(h) $f(x) = (1-x)^{\frac{1}{x}}$

2. Find
- y'
- given that

$$x + 2y = \tan(x^3 y).$$

3. Use implicit differentiation to find
- $\frac{dy}{dx}$
- .

(a) $y = e^{x^5 y}$

(b) $(2x + y)^{\frac{3}{2}} = x^2 + y^2$

(c) $\sin^2(x + y) = x + 4y$

4. Given that
- $x^3 + y^3 = 12$
- , prove that

$$\frac{d^2 y}{dx^2} = -\frac{24x}{y^5}.$$

5. The ideal gas law

$$PV = nRT$$

relates the pressure P in kPa (kiloPascals), volume V in L (litres), temperature T in K (kelvins), and number of moles n of a gas, where R is a constant. For a certain gas, we are given that $nR = 5$, and so the gas is described by the equation

$$PV = 5T$$

If we heat the gas at a rate of 2 K/s and compress it at a rate of 0.1 L/s, what is the rate of change of the pressure of the gas when the temperature is 300 K and the volume is 10 L?

6. An airplane is flying at an altitude of 10km and passes directly over the radar antenna. At some moment later, the radar detects that the distance to the plane is 26 km and this distance is increasing at a rate of 480 km/h. What is the speed of the plane at this instant?
7. A spherical balloon is being inflated so that the volume increases at a constant rate. Find this rate if the surface area of the balloon is increasing at a rate of 20 cm²/s when the radius is 10 cm.

$$\left[\text{Volume of a sphere: } V = \frac{4}{3}\pi r^3, \text{ Surface area of a sphere: } S = 4\pi r^2 \right].$$

8. A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at the rate of 3 feet per second.
- (a) How fast is the top of the ladder moving down the wall when the top is 20 feet above the ground?
- (b) Find the rate at which the angle between the ladder and the ground is changing at the same moment.