

Name: \_\_\_\_\_

Due: Friday, 22nd November, 2002

Student No.: \_\_\_\_\_

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ASSIGNMENT 6

Mathematics 1000

FALL 2002

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1. Given  $f(x) = x^5 - 2x^4$ ,
  - (a) Find the critical numbers, and the intervals on which  $f(x)$  is increasing, decreasing, concave up, and concave down. Find the exact coordinates of all extreme points, inflection points and intercepts.

(b) Note that  $f''(0) = 0$ . Is  $(x, y) = (0, 0)$  an inflection point? Why or why not?

2. Determine the constants  $a$ ,  $b$ , and  $c$  such that the graph of the function

$$f(x) = x^3 + ax^2 + bx + c$$

passes through the point  $(0, 1)$ ,  $x = 3$  is a critical number of  $f(x)$ , and there is an inflection point at  $x = \frac{7}{6}$ .

3. Evaluate each limit. If the limit does not exist, explain why.

(a)  $\lim_{x \rightarrow \infty} \frac{3x^2 - x - 2}{5x^2 + 4x + 1}$

(b)  $\lim_{x \rightarrow \infty} \frac{4}{2 - 5e^{-x}}$

(c)  $\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2 + 1}}{3x - 5}$

(d)  $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 1} - x)$

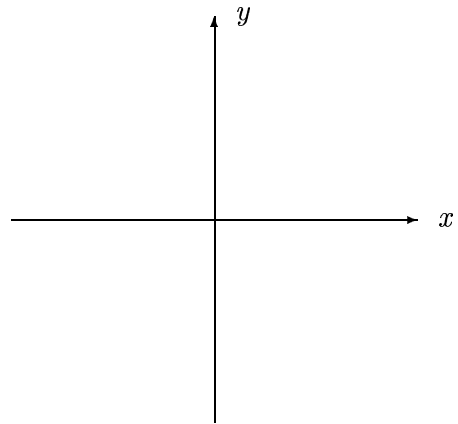
4. Identify all intercepts, asymptotes (horizontal and vertical), critical points, points of inflection and relative extrema of the function

$$f(x) = \frac{10x - 10}{x^2 - 2x - 8}$$

Also, determine the intervals on which the function is increasing, decreasing, concave up and concave down.

5. Sketch the graph of a function  $y = f(x)$  with the following properties, on the axes provided.

- $f'(x) < 0$  for  $x < 1$ ,  $3 < x < 4$  and  $x > 4$
- $f'(x) > 0$  for  $1 < x < 3$
- $f''(x) < 0$  for  $x < 0$  and  $2 < x < 4$
- $f''(x) > 0$  for  $0 < x < 2$  and  $x > 4$
- $f'(1) = f'(3) = 0$
- $f''(0) = f''(2) = 0$
- $\lim_{x \rightarrow 4^-} f(x) = -\infty$ ,  $\lim_{x \rightarrow 4^+} f(x) = \infty$
- $\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0$
- $f(0) = -\frac{3}{2}$ ,  $f(1) = -\frac{5}{2}$ ,  $f(2) = 0$ ,  
 $f(3) = 4$ ,  $f\left(\frac{7}{2}\right) = 0$



6. A rectangular storage container with an open top is to have a volume of  $10 \text{ m}^3$ . The length of its base is twice the width. Material for the base costs \$10 per square metre. Material for the side costs \$6 per square metre. Find the cost of materials for the cheapest such container.

7. A man is at point  $A$  on a bank of a straight river, 3 km wide, and wants to reach point  $B$ , 8 km downstream on the opposite bank, as quickly as possible. He could row his boat directly across the river to point  $C$  and run to  $B$ , or he could row directly to  $B$ , or he could row to some point  $D$  between  $C$  and  $B$  and then run to  $B$ . If he can row at 6 km/hr and run at 8 km/hr, where should he land to reach  $B$  as soon as possible?

8. Find the dimensions of the largest isosceles triangle that can be inscribed in a circle of radius 4.