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Reaching Out Vol. 1, No. 2
Mathematics Students
Lewisporte Collegiate

To whom it may concern:

Hopefully if you are reading this it means that you enjoyed the first installment of this newsletter and have come back for more mathematics. Just a quick re–introduction if this is your first installment of the newsletter. I was a student at Lewisporte Regional High School and a graduate of 1992. I attended Memorial University and studied mathematics and then went on to Simon Fraser University in British Columbia and completed both a MSc and PhD in applied and computational mathematics. Since July 2004 I have been a faculty member in the department of mathematics and statistics at Acadia University in Nova Scotia.

In this installment and the next I will introduce sequences and series, along the way you will be introduced to *sigma* notation and ideas of convergence and recursions relations to name a few. At the end of this letter you will hints and solutions for the problems from the previous installment.

Please do not hesitate to contact me at *ronald.haynes@acadiau.ca* should you have questions. You can also visit my website *http://rhaynes.acadiau.ca/personal* to find out more info about my work and teaching as well as links to our department and Acadia. Link to previous installments of this newsletter may be found at this website as well.

Sequences and Series Part I

A sequence is simply a list of numbers with order. For example

$$\{2, 4, 6, 8, 10\}$$

is a finite sequence consisting of 5 terms or numbers. To refer to particular terms in a sequence we often use a subscript system. We will use t_n to denote the nth term in the sequence. So in the example above the third term, t_3 is 6, and $t_5 = 10$ and so on. The terms in a sequence do not necessarily follow a pattern but often they do. The example above consists of the first five even integers, in general we see that $t_n = 2n$ for $n = 1, 2, \ldots, 5$.

The sequence

$$\{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots\}$$

is a very famous sequence known as the Fibonacci sequence, named after the famous Italian mathematician who lived from 1180–1250. From the third term on each term is the sum of the previous two terms, $t_3 = t_2 + t_1$, $t_4 = t_3 + t_2$, so in general

$$t_n = t_{n-1} + t_{n-2}$$

where we have to specify two starting conditions $t_1 = 1$ and $t_2 = 1$. The statement $t_n = t_{n-1} + t_{n-2}$ is an example of a *recursion relation.*

Two particularly common types of sequences are known as arithmetic and geometric sequences. Arithmetic sequences have the property that the difference between consecutive terms is constant while geometric sequences have a common ratio between consecutive terms.

If we denote the first term of an arithmetic sequence by a and if d is the common difference between terms then the nth term t_n is given by the formula $t_n = a + (n-1)d$ for $n = 1, 2, \ldots$. Geometric sequences with first term a and common ratio r are specified by the formula $t_n = ar^{n-1}$ for $n = 1, 2, \ldots$.

Examples. Consider the arithmetic sequence

$$\{2, 5, 8, 11, \ldots\}.$$

The nth term of this sequence is given by $t_n = 2 + 3(n-1)$ for $n = 1, 2, \ldots$. In general the formula for the nth term of an arithmetic sequence is given by a linear function of n. The nth term of the geometric sequence

$$\{2, 1, 1/2, 1/4, \ldots\}$$

is given by $t_n = 2\left(\frac{1}{2}\right)^{n-1}$ for $n = 1, 2, \ldots$.

If we add the terms of sequence we obtain a *series.* Series are finite if we add up a finite number of terms — this is always possible! Series are infinite if we (attempt to) add up an infinite number of terms. If this doesn't sound easy you are right! I would have more to say about infinite series next month. In this letter we will focus on finite sums. We will denote the sum of the first n terms of a sequence by s_n, that is

$$s_n = t_1 + t_2 + \cdots + t_n.$$

Mathematics often use a short–hand notation to represent sums, for example s_n may be rewritten as

$$s_n = \sum_{i=1}^{n} t_i.$$

Here $\sum$ is the Greek capital letter Sigma corresponding to S in the English language, it tells you that you are forming a sum. In this case the notation says that we are up to add up the t_i's where i goes from 1 to n over the positive integers. For example

$$\sum_{i=1}^{5} 2i - 1 = (2(1)-1) + (2(2)-1) + (2(3)-1) + (2(4)-1) + (2(5)-1) = 1 + 3 + 5 + 7 + 9.$$

This is an *arithmetic series* since we are adding up an arithmetic sequence.

There are some simple formulas available for the sum of the first n terms of arithmetic and geometric sequences, these are often referred to as formulas for the sum of finite arithmetic and geometric series. Do a few google searches and locate these formulas and their derivations.

Here are a few problems for you think about regarding this months material. After the problems are hints and solutions for last months problems.

Problems for Vol. 1 No. 2

Problem 1. The sum of $200 - 199 + 198 - 197 + 196 - \cdots + 2 - 1$ is what?

Problem 2. The sum of the first n terms of a sequence is $n(n+1)(n+2)$. What is the 10th terms of the sequence?

Problem 3. The three positive numbers a, b and c form a geometric sequence. Prove that the numbers $\log a$, $\log b$ and $\log c$ form an arithmetic sequence.

Problem 4. If a, b and c form an arithmetic sequence, show that

$$(b-c)x^2 + (c-a)x + (a-b) = 0$$

has equal roots.

Problem 5. For a geometric sequence of positive terms to have any term equal to the sum of the next two terms, what must the common ratio be?

Hints and Brief Solutions for Vol. 1 No. 1

Problem 1. If r and s are roots of $ax^2 + bx + c = 0$ then find a value of

$$\frac{1}{r^2} + \frac{1}{s^2}$$

in terms of a, b and c.

Solution.

$$\frac{1}{r^2} + \frac{1}{s^2} = \frac{r^2 + s^2}{r^2 s^2} = \frac{(r+s)^2 - 2rs}{(rs)^2}$$

Now use the formulas for $r + s$ and rs in terms of a, b and c.

Problem 2. If r and $2r$ are roots of $ax^2+bx+c = 0$ then a, b and c are related by what expression?

Solution.

So $x^2+b/ax+c/a = (x-r)(x-2r) = x^2-3rx+2r^2$. Comparing coefficients we have $-3r = b/a$ and $2r^2 = c/a$. The first equation tells us $r = -b/(3a)$. Substituting this into the 2nd equation gives

$$2\left(\frac{-b}{3a}\right)^2 = \frac{c}{a},$$

or $2b^2 = 9ac$ which is the required relation.

Problem 3. If r and s are roots of $x^2 + x + 7$ then compute the value of

$$2r^2 + rs + s^2 + r + 7$$

without actually finding r and s.

Hint.

Find formulas for $r + s$ and rs using the coefficients of the quadratic as above. Then rewrite the given expression in terms of $r + s$ and rs.

Sincerely,

Dr. Ronald D. Haynes