

Mathematics 2260: ODE (I)

Assignment 5, due day Nov 27 at classroom

1. Find the Laplace transform of the following functions(show your work).

1) t

2) te^{-at}

3) $f(t) = \begin{cases} 1, & 0 \leq t < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} \leq t < \infty \end{cases}$

2. Find the inverse Laplace transform of the given functions

a) $\frac{4}{s^2+4s-5}$

b) $\frac{4s+4}{s^2+2s+5}$

c) $\frac{5!}{(s-2)^4}$

d) $\frac{2(s-1)}{s^2-2s+2}e^{-2s}$

3. Using Laplace transform to solve the following equation

a) $y'' + 2y' + 5y = 0, \quad y(0) = 2, y'(0) = -1.$

b) $y^{(4)} - 4y''' + 6y'' - 4y' + y = 0, \quad y(0) = 0, y'(0) = 1, y''(0) = 0, y'''(0) = 1.$

c) $y'' + y = f(t), \quad y(0) = 0, y'(0) = 1, \quad f(t) = \begin{cases} 1 & 0 \leq t < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \leq t < \infty \end{cases}.$

d) $y'' + 3y' + 2y = u_2(t), \quad y(0) = 0, y'(0) = 1.$

4. Transform the given equation into a system of first order equations

a) $u'' + \frac{1}{2}u' + 2u = 0$

b) $u'' - \frac{1}{2}u' + 2u = 4 \cos t$

c) $t^2u'' + tu' + (t^2 - 1)u = 0$

d) $u^{(3)} - u = 0.$

5. Solve the following system

a)
$$\begin{cases} x_1' = -2x_1 + x_2 \\ x_2' = x_1 - 2x_2 \\ x_1(0) = 1, x_2(0) = 1 \end{cases}$$

b)
$$\begin{cases} x_1' = -\frac{1}{2}x_1 + 2x_2 \\ x_2' = -2x_1 - \frac{1}{2}x_2 \end{cases}, \quad x_1(0) = 2, x_2(0) = 2.$$

6. Using Laplace transform to Solve the above two systems in Question 5.