Mathematics 2260: ODE (I)　　　Fall 2017

Assignment 4**, due day Nov 9 at classroom**

1. Find the solutions of the given differential equations

$$1)\ y'' + 9y = 4t^2e^{3t} + 2 \qquad\qquad 2)\ y'' - 2y' - 3y = -te^{-t}$$

$$3)\ y'' + 4y = \cos 2t, \qquad y(0) = 2,\ y'(0) = 1$$

2. Without solving the equation, determine a suitable form for particular solution $Y(t)$.

$$1)\ y'' + 3y' + 2y = e^{-t}(t+3)\sin 2t + e^t \sin 3t + 10$$

$$2)\ y'' + 2y' = 2t + \cos t + e^{-2t}t^3.$$

3. Suppose the given functions y_1 and y_2 satisfy the corresponding homogeneous equation. Using the method of variation of parameters to solve the non-homogeneous equations

$$1)\ t^2y'' - t(t+2)y' + (t+2)y = 2t^3, t > 0, \qquad y_1(t) = t,\ y_2(t) = te^t$$

$$2)\ ty'' - (1+t)y' + y = t^2e^{2t},\ t > 0, \quad y_1(t) = 1 + t,\ y_2(t) = e^t$$

4. Suppose that $y_1(t) = t^2$ is a solution of equation

$$t^2y'' - 2y = 0,\ t > 0.$$

a) Using the method of reduction of order to find a second solution $y_2(t)$.

b) Using the method of variation of parameters to solve the following equation

$$t^2y'' - 2y = 3t^2 - 1.$$

5. Determine whether the given set of functions is linearly independent or linearly dependent.

$$1)\ 12t - 3,\ t^2 + 1,\ 2t^2 - t \qquad\qquad 2)\ 1,\ \cos t,\ \sin t$$

6. Find the general solution of the given differential equations

$$1)\ y''' - y'' - y' + y = 0 \qquad\qquad 2)\ y^{(4)} - y'' = 0.$$

$$3)\ y''' - y'' + y' - y = 2e^t + 3, \qquad 4)\ y^{(4)} - y = 3t + \sin t$$