Mathematics 2260: ODE (I) Fall 2017

Assignment 2, **due day Oct 5 (4:00pm)**

1. Find the solutions by using substitution method

$$1)\ y' = (x+2y)/(2x+y) \quad 2)\ (y^4 - 2x^3y)dx + (x^4 - 2xy^3)dy = 0$$

2. Find the solution of differential equations by using appropriate method:

$$1)\ y'' + \frac{1}{t}y' = 0 \qquad 2)\ t^2y' + 2ty - y^3 = 0 \quad 3)\ y' = ry - ky^2$$

3. suppose that a certain population obeys the logistic equation $dy/dt = ry[1 - y/K]$.

(a) If $y_0 = K/3$, find the time τ at which the initial population has doubled. Find the value of τ corresponding to $r = 0.025$ *per year*.

(b)If $y_0 = \alpha K$, find the time T at which $y(T) = \beta K$, where $0 < \alpha, \beta < 1$.

4. A colony of rabbits is introduced on a large island. Initially, there are 10 rabbits per hectare. In the absence of predators, the population grows in proportion to its current size. However, hunters are allowed to travel to the island and trap H rabbits per hectare each year. The density of the rabbit population $y(t)$ can therefore be represented by the equation

$$\frac{dy}{dt} = ry - H,$$

where r is a positive constant.

1) If no hunting was allowed, there would be 45 rabbits per hectare after 3 years. Use this information to determine r.

2) In fact, because of the hunting, there are only 17 rabbits per hectare after 3 years. Find H.

3) Assuming that hunting continues at this rate, determine the number of rabbits per hectare that will survive on the island after 10 years.

5. Determine (with out solving the problem) an interval in which the solution of the given initial value problem is certain to exist:

$$t(t-4)y' + y = 2t,\ y(2) = 1.$$

6.Determine whether or not each of the following equation is exact. If it is exact, find the general solution; otherwise, find an integrating factor and solve it:

$$1)\ (2xy^2 + 2y) + (2x^2y + 2x)y' = 0 \quad 2)\ (e^x \sin y - 2y \sin x)dx + (e^x \cos y + 2\cos x)dy = 0$$

$$3)\ \frac{dy}{dx} = -\frac{ax+by}{bx+cy} \qquad 4)\ \ ydx + (2xy - e^{-2y})dy = 0$$

7. Determine the longest interval in which the given value problem is certain to have twice differentiable solution. Do not attempt to find the solution.

$$1)\ (x-3)y'' + xy' + (\ln|x|)y = 0,\ y(1) = 0, y'(1) = 1$$

$$2)\ (x-2)y'' + y' + (x-2)(\tan x)y = 0,\ y(3) = 1, y'(3) = 2.$$