Mathematics 2260: ODE (I) Fall 2017

Assignment 1, **due day Sep 26 at classroom**

1.Determine the order of the following equations; also state whether the equation is linear or nonlinear.

$$(a)\ \frac{d^4y}{dt^4} + 3t^4\frac{d^2y}{dt^2} + y = \sin t^4; \qquad (b)\ \frac{d^2y}{dt^2} + \cos(t+y) = t^2$$

$$(c)\ u_{xx} + u_y + u_{yyyx} = 0; \qquad (d)\ u_t + uu_x = 1 + u_{xxx}$$

2. Verify that $y_1 = e^{-3t}$ and $y_2 = e^t$ are the solutioms of the following equation

$$y'' + 2y' - 3y = 0, t > 0.$$

3. Consider a population p of field mice that grows at a rate propotional to the current population:

$$\frac{dp}{dt} = rp \quad \text{where } r \text{ is a constant.}$$

(a) Assume that $p(0) = p_0$. Find the rate r so that the population doubles in 30 days.

(b)Find r if the population doubles in N days.

4. Find the general solution of the following problem and use it to determine how the solutions behave as $t \to \infty$:

$$(a)\ y' + 3y = 1 + e^{-2t} \quad (b)\ y' - 2y = t^2e^{2t} \qquad (c) y' + y = te^{-t} + 1$$

5. Solve the equation: $t^3y' + 4t^2y = e^{-t}, y(-1) = 0, t < 0$ by the method of integrating factor.

6. Find the solutions to the following initial problem

(a) $y' + 2y = te^{-2t},\ y(1) = 0$; (b) $y' + \frac{2}{t}y = \cos(t)/t^2,\ y(\pi) = 0$

(c) $y' - 2y = e^{2t},\ y(0) = 2, t > 0.$

7. Find the general solutions of the following equations by the method of seperation of variables.

(a) $\frac{dy}{dx} = \frac{x-e^{-x}}{y+e^y}$ (b) $y' = 2\sin(2x)/(3+2y)$, $(c)\ xy' = (1-y^2)$

(d) $y' = 2x/(1+2y),\ y(2) = 0.$

8. Find the value of y_0 for which the solution of the initial value problem

$$y' - y = 1 + te^{-t}, \quad y(0) = y_0$$

remains finite as $t \longrightarrow \infty$