

*for student practice only*

Note that the final exam also covers material after HW8, including, for instance, calculating determinant by row operations, eigenvalues and eigenvectors, similarity and diagonalization etc. Although HW9 is not graded, you should work on it as serious as other homework assignments in order to be familiar with the above mentioned material. )

1. Find the determinant by reducing to triangular form for the following matrices.

$$(a) A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & 1 & 4 \\ 1 & -1 & 5 \end{bmatrix}.$$

$$(b) A = \begin{bmatrix} 1 & -1 & 2 & -2 \\ 2 & 5 & 3 & 1 \\ -1 & 0 & 2 & -1 \\ 3 & 1 & 2 & 0 \end{bmatrix}.$$

2. Find the characteristic polynomial, the real eigenvalues and the corresponding eigenspaces of each of the following matrices:

$$(a): A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}.$$

$$(b): A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & -6 \\ 1 & 2 & -1 \end{bmatrix}.$$

3. Are the following matrices  $A$  and  $B$  similar to each other?

$$(a): A = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & 4 \\ 3 & 3 \end{bmatrix}.$$

$$(b): A = \begin{bmatrix} 1 & -4 \\ -2 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}.$$

4. Find a matrix  $P$  such that  $P^{-1}AP$  is a diagonal matrix.

$$(a): A = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}.$$

$$(b): A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}.$$

5.  $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ . Find a matrix  $P$  such that  $P^{-1}AP = D$  is a diagonal matrix. Find  $D^{16}$ .