

MATH2050 Assignment 8

Due: Wednesday 22 November

[10] 1. For each of the following matrices,

- find the matrix M of minors, the matrix C of cofactors, the adjoint $\text{adj}A$, and compute $(\text{adj}A)A$ and $A(\text{adj}A)$.
- find $\det A$.
- find A^{-1} or state why it doesn't exist.

$$(a) A = \begin{bmatrix} 4 & 7 \\ 8 & 14 \end{bmatrix} \quad (b) A = \begin{bmatrix} 3 & 5 & 2 \\ 4 & 8 & 9 \\ -1 & 2 & 5 \end{bmatrix}$$

[6] 2. Use a Laplace expansion to find the determinant of each of the following matrices.

$$A = \begin{bmatrix} 4 & 3 & 1 \\ 0 & 2 & 1 \\ 3 & 5 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 1 & 2 & 0 & 1 \\ -1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

[6] 3. Are the following sets of vectors linearly independent or not?

$$(a) \vec{a}_1 = \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}, \vec{a}_2 = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix} \text{ and } \vec{a}_3 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$$
$$(b) \vec{b}_1 = \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \vec{b}_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \vec{b}_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} \text{ and } \vec{b}_4 = \begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

[6] 4. Let A and B be 4×4 invertible matrices such that $\det A = -2$ and $\det B = 5$. Find the following determinants

- $\det A^{-1}$
- $\det \frac{1}{3}A$
- $\det A^{-1}B^T A^2(2B)$.

$$[4] 5. \text{ Suppose } \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 5. \text{ Find } \begin{vmatrix} 2p & 7a & a-x \\ 2q & 7b & b-y \\ 2r & 7c & c-z \end{vmatrix}.$$

[8] 6. Without using the fact that $\det(AB) = (\det A)(\det B)$, show that $\det(EA) = (\det E)(\det A)$ for any $n \times n$ matrix A and elementary matrix E . (Consider separately the cases when A is singular and invertible).