MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 7 **Mathematics 2050** **fall 2017**

Due:November 7, 2017 . SHOW ALL WORK

[4] 1. Recall that a circle in the xy-plane has an equation of the form $x^2+y^2+ax+by+c=0$. Find the equation of the circle that passes through the points $(10,7)$, $(-4,-7)$ and $(-6,-1)$. Complete squares to find the centre and the radius of the circle.

[3] 2. Calculate AB and BA and determine whether A and B are inverses of each other.

(a) $A = \begin{bmatrix} 2 & 0 & -\frac{1}{2} \\ -1 & 0 & \frac{1}{2} \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ -1 & -2 \\ 2 & 4 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ and $B = \frac{1}{24}\begin{bmatrix} 24 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$

[4] 3. Find all values of a for which the following homogeneous system has a nontrivial solution. Also, find all solutions.

$$\begin{cases} x - \ \ y - 2z &= 0 \\ x - 2y + az &= 0 \\ 2x + ay - 5z &= 0 \end{cases}$$

[5] 4. For the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$, find A^{-1} if it exists.

[3] 5. Given the matrices $A = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 4 & 2 \end{bmatrix}$, find a matrix X such that $A^{-1}XA = B$.

[4] 6. Given the matrix $C = \begin{bmatrix} -1 & 1 \\ 4 & 2 \end{bmatrix}$, find a matrix X such that $(X^T - 2I)^{-1} = C$

[3] 7. (a) If A and B are invertible $n \times n$ matrices that commute, prove that B and A^{-1} commute.

[4] (b) Let A be an $n \times n$ matrix such that $A^2 + 2A + I = 0$. Prove that A is invertible and find its inverse.

[4] (c) Let $A + I$ be invertible. Show that $(A+I)^{-1}$ and $(I - A)$ commute.

[6] 8. Let $A = \begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$.

(a) (a) Express the matrix A as the product of elementary matrices.

(b) (b) Express the matrix A^{-1} as the product of elementary matrices.

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[40]