MATH 2050 **Assignment 6** **Fall 2017**

Due: Wed, November 1

[5] 1. For what value of c does

$$\begin{aligned} x+y+2z &= 2 \\ -x+y+z &= c \\ 4x \quad +2z &= 2 \end{aligned}$$

have a solution? Is it unique?

[20] 2. Write all solutions of the following linear systems in vector form.

(a)

$$\begin{aligned} x_1+2x_2-x_3+3x_4 &= 4 \\ x_2 \quad +2x_4 &= 1 \\ x_1+x_2 \quad -x_4 &= 3 \end{aligned}$$

(b)

$$\begin{aligned} x+2y+4z &= 3 \\ x+2y+6z &= 5 \\ x+3y+5z &= 4 \end{aligned}$$

(c)

$$\begin{aligned} x+5y-2z &= -2 \\ 3x+15y-6z &= -6 \\ -x-5y+2z &= 2 \end{aligned}$$

(d)

$$\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -2 & 4 & -2 & 0 & 0 \\ 0 & -3 & 6 & -3 & 0 \\ 0 & 0 & 4 & -8 & 4 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

[5] 3. Find conditions on a, b, and c (if any) such that the system

$$\begin{aligned} x \quad + z &= -1 \\ 2x-y \quad &= 2 \\ y+2z &= -4 \\ ax+by+cz &= 3 \end{aligned}$$

has (i) no solution, (ii) a unique solution, and (iii) infinitely many solutions.

[10] 4. Find all solutions of

$$\begin{aligned} x+3y \quad\quad &= 0 \\ x+4y+\ \ 5z &= 0 \\ 2y+10z &= 0 \end{aligned}$$

Does

$$\begin{aligned} x+3y \quad\quad &= \pi \\ x+4y+\ \ 5z &= \sqrt{17/19} \\ 2y+10z &= e^{-\sqrt{2}} \end{aligned}$$

have a unique solution?

[5] 5. Show that the vectors $\vec{u} = \begin{bmatrix} 1 \\ 3 \\ -1 \\ -2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 2 \\ 6 \\ 0 \\ 4 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 6 \end{bmatrix}$ are linearly independent.

[5] 6. For what values of x (if any) are the vectors $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -1 \\ -1 \\ x-3 \end{bmatrix}$, and $\vec{w} = \begin{bmatrix} 1 \\ -1 \\ 3x^2-3 \end{bmatrix}$ linearly independent?

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[50]