

Due: Oct. 25

[5] 1. Write down the 2×3 matrix $A = [a_{ij}]$ with $a_{ij} = ij - \cos \frac{\pi j}{3}$.

[12] 2. Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$.

i) Compute AB , $(AB)^T$, $A^T B^T$ and $B^T A^T$. Do you have $(AB)^T = B^T A^T$?

ii) Compute $(A + B)^2$ and $A^2 + 2AB + B^2$. Are these equal? What is the correct expansion of $(A + B)^2$?

iii) Compute $AB - 2B$ and $(A - 2I_2)B$. Are they equal?

[5] 3. Find the matrix A if: $\left(3A^T + 2 \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\right)^T = \begin{bmatrix} 8 & 0 \\ 3 & 1 \end{bmatrix}$

[6] 4. Compute the following matrix products.

$$(a) \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}$$

$$(b) \begin{bmatrix} 5 & 0 & -7 \\ 1 & 5 & 9 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 1 & 9 & 7 \\ -1 & 0 & 2 \end{bmatrix}$$

[6] 5. Give an example with two matrices A and B such that $AB = 0$ does not imply that $A = 0$ or $B = 0$.

[5] 6. Express the system

$$\begin{cases} x + 10z = 5, \\ 3x + y - 4z = -1, \\ 4x + y + 6z = 1. \end{cases}$$

in the form of $AX = b$.

[6] 7. Write $-2 \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 7 \end{bmatrix} + 3 \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$ in the form of AX for a suitable matrix A and vector X .

[5] 8. Suppose a matrix A satisfies $A = 2A^T$. Show that necessarily $A = 0$.