MATH 2050 **Assignment 5** **Fall 2017**

Due: Oct. 25

[5] 1. Write down the 2×3 matrix $A = [a_{ij}]$ with $a_{ij} = ij - \cos \frac{\pi j}{3}$.

Solution: $A = \begin{bmatrix} 1/2 & 5/2 & 4 \\ 3/2 & 9/2 & 7 \end{bmatrix}$.

[12] 2. Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$.

i) Compute $AB, (AB)^T, A^TB^T$ and B^TA^T. Do you have $(AB)^T = B^TA^T$?

Solution: $AB = \begin{bmatrix} 1 & 6 \\ 2 & 7 \end{bmatrix}$, $(AB)^T = \begin{bmatrix} 1 & 2 \\ 6 & 7 \end{bmatrix}$, $A^TB^T = \begin{bmatrix} 7 & 2 \\ 6 & 1 \end{bmatrix}$

and $B^TA^T = \begin{bmatrix} 1 & 2 \\ 6 & 7 \end{bmatrix}$.

Therefore we have $(AB)^T = B^TA^T$ not $(AB)^T = A^TB^T$

ii) Compute $(A+B)^2$ and $A^2 + 2AB + B^2$. Are these equal? What is the correct expansion of $(A+B)^2$?

Solution: $(A+B)^2 = \begin{bmatrix} 16 & 24 \\ 8 & 16 \end{bmatrix}$.

$A^2 + 2AB + B^2 = \begin{bmatrix} 10 & 24 \\ 8 & 22 \end{bmatrix}$.

They are not equal. The correct answer is $(A+B)^2 = A^2 + AB + BA + B^2$.

iii) Compute $AB - 2B$ and $(A - 2I_2)B$. Are they equal?

solution: $AB - 2B = \begin{bmatrix} -1 & 0 \\ 2 & 5 \end{bmatrix}$. $(A - 2I_2)B = \begin{bmatrix} -1 & 0 \\ 2 & 5 \end{bmatrix}$.

Yes, they are equal.

[5] 3. Find the matrix A if : $\left(3A^T + 2\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\right)^T = \begin{bmatrix} 8 & 0 \\ 3 & 1 \end{bmatrix}$

Solution: $\left(3A^T + 2\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\right)^T = \begin{bmatrix} 8 & 0 \\ 3 & 1 \end{bmatrix} \Longrightarrow$

$(3A^T)^T + \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}^T = \begin{bmatrix} 8 & 0 \\ 3 & 1 \end{bmatrix}$

$3A + \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 3 & 1 \end{bmatrix}$

$3A = \begin{bmatrix} 6 & 0 \\ 3 & -3 \end{bmatrix} \Rightarrow A = \frac{1}{3}\begin{bmatrix} 6 & 0 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$.

OR: We can obtain $3A^T + \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 3 & 1 \end{bmatrix}^T = \begin{bmatrix} 8 & 3 \\ 0 & 1 \end{bmatrix} \Longrightarrow$

$$3A^T = \begin{bmatrix} 6 & 3 \\ 0 & -3 \end{bmatrix} \qquad A^T = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}, \Longrightarrow A = \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$$

[6] 4. Compute the following matrix products.

$(a) \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}$ $\qquad$ $(b) \begin{bmatrix} 5 & 0 & -7 \\ 1 & 5 & 9 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 1 & 9 & 7 \\ -1 & 0 & 2 \end{bmatrix}$

Solution: (a) $\begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 3 \\ -2 & -1 & -3 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 0 & -7 \\ 1 & 5 & 9 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 1 & 9 & 7 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 17 & 15 & -9 \\ -2 & 48 & 54 \end{bmatrix}$.

[6] 5. Give an example with two matrices A and B such that $AB = 0$ does not imply that $A = 0$ or $B = 0$.

For example, $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $\Longrightarrow AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

AB is the zero matrix, but neither A or B is zero matrix.

[5] 6. Express the system

$$\begin{cases} x + 10z = 5, \\ 3x + y - 4z = -1, \\ 4x + y + 6z = 1. \end{cases}$$

in the form of $AX = b$.

Solution: $A = \begin{bmatrix} 1 & 0 & 10 \\ 3 & 1 & -4 \\ 4 & 1 & 6 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $b = \begin{bmatrix} 5 \\ -1 \\ 1 \end{bmatrix}$

[6] 7. Write $-2\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} + 0\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 4 \\ 2 \\ 7 \end{bmatrix} + 3\begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$ in the form of AX for a suitable matrix A and vector X.

Solution: $A = \begin{bmatrix} 1 & 2 & -3 & 4 & 6 \\ 1 & 0 & 1 & 2 & 5 \\ -2 & 1 & 1 & 7 & 4 \end{bmatrix}$, $X = \begin{bmatrix} -2 \\ 0 \\ 1 \\ -1 \\ 3 \end{bmatrix}$

[5] 8. Suppose a matrix A satisfies $A = 2A^T$. Show that necessarily $A = 0$.

Solution: Suppose $A = 2A^T$. We need to show that $A = 0$.

Now, $A = 2A^T = 2[2A^T]^T = 4[(A^T)^T] = 4A$. Hence, $3A = 0$. So, $A = \frac{1}{3}(3A) = \frac{1}{3}(0) = 0$

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