MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATH2050 Assignment 4

Due: Wednesday 11 October

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[6] 1. Let ℓ_1, ℓ_2, ℓ_3 and ℓ_4 be lines given by the following equations.

$$\ell_1\colon \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + t_1 \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \qquad \ell_2\colon \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ 3 \end{bmatrix} + t_2 \begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix}$$

$$\ell_3\colon \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + t_3 \begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix} \qquad \ell_4\colon \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \\ -1 \end{bmatrix} + t_4 \begin{bmatrix} -6 \\ -4 \\ 6 \end{bmatrix}$$

Determine whether the following pairs of lines are parallel, skew, or if they intersect. In each case explain your answer and find the intersection if it exists.

(a) ℓ_1 and ℓ_2. (b) ℓ_1 and ℓ_3. (c) ℓ_2 and ℓ_4.

[6] 2. Let ℓ be the line with equation $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} + t \begin{bmatrix} -4 \\ 1 \\ 3 \end{bmatrix}$.

(a) Find the distance from $P(2, -1, 5)$ to ℓ.

(b) Find the point of ℓ closest to P.

[6] 3. (a) Find two orthogonal vectors in the plane π given by the equation $2x - 3y + 4x = 0$.

(b) Given the vector $\vec{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, find a vector $\vec{p}$ on the plane π such that $\vec{w} - \vec{p}$ is orthogonal to every vector on π.

[4] 4. Let $\vec{u} = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$. Find $\text{proj}_{\vec{u}}\vec{v}$ and $\text{proj}_{\vec{v}}\vec{u}$.

[6] 5. Given $P(1, 1, 3)$ and a plane π with equation $x + 3y - z = 7$.

(a) Find the distance from point P to the plane π.

(b) Find the point on π closest to P.

[4] 6. Describe in geometrical terms the set of all points one unit away from the plane with equation $2x - 2y + z = 3$. Find an equation for this set of points.

[2] 7. Suppose vectors $\vec{u}$ and $\vec{v}$ are linearly independent. Show that the vectors $\vec{w} = \vec{u} - \vec{v}$ and $\vec{z} = \vec{u} + 3\vec{v}$ are also linearly independent.

[6] 8. Determine whether or not the following sets of vectors are linearly independent or linearly dependent. If the vectors are linearly dependent then give an example of a nontrivial linear combination of the vectors which equals the zero vector.

(a) $\vec{u} = \begin{bmatrix} 2 \\ 4 \\ 3 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 5 \\ 6 \\ 9 \end{bmatrix}$ (b) $\vec{u} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \quad \vec{v} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 7 \\ -3 \\ 1 \end{bmatrix}$ (c) $\vec{e_1}, \vec{e_2}, \vec{e_3}, \vec{0}$

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