MATH 2050 **Assignment 2** **Fall 2017**

Due: Wed, September 27

[5] 1. Let $\vec{u} = \begin{bmatrix} -3 \\ 4 \\ 0 \\ 12 \end{bmatrix}$. Find a unit vector in the direction of $\vec{u}$ and a vector of norm 4 in the direction opposite to $\vec{u}$.

[5] 2. Find all vectors $\vec{u}$ that are parallel to $\vec{v} = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$ and satisfy $\|\vec{u}\|^2 = 2\|\vec{v}\|^2$.

[10] 3. (a) Let $\vec{u}$ and $\vec{v}$ be vectors of magnitude 2 and 5, respectively, and suppose that $\vec{u} \cdot \vec{v} = -3$. Find $(\vec{u} - \vec{v}) \cdot (2\vec{u} - 3\vec{v})$ and $\|\vec{u} + \vec{v}\|$.

(b) The two vectors $3\vec{u} + \vec{v}$ and $\vec{u} - 4\vec{v}$ are perpendicular. Find the angle between $\vec{u}$ and $\vec{v}$ if $\|\vec{u}\| = 2\|\vec{v}\|$.

[5] 4. Let $\vec{u} = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 2k - 3 \\ 3k - k^2 \\ 3 \end{bmatrix}$. Determine all values of k for which $\vec{u}$ and $\vec{v}$ are orthogonal.

[5] 5. Find all real numbers x such that $\vec{u} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ x \\ 2 \end{bmatrix}$ are at an angle of $\frac{\pi}{3}$.

[5] 6. Let $\vec{u} = \begin{bmatrix} 3 \\ -6 \\ 3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

(a) Show that $\vec{u}$ is orthogonal to $\vec{v} - \vec{w}$.

(b) Show that $\vec{u}$ is orthogonal to $a\vec{v} + b\vec{w}$ for any scalars a and b.

[5] 7. Let $\vec{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\vec{v}$ be a unit vector in the plane. What are the possible values of $\|\vec{u} + \vec{v}\|$? Give a unit vector $\vec{v}$ such that $\|\vec{u} + \vec{v}\| = \sqrt{3}$.

[5] 8. Give vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ such that $\vec{u} \cdot \vec{v} = 0$ and $\vec{v} \cdot \vec{w} = 0$, but $\vec{u} \cdot \vec{w} \neq 0$.

[5] 9. Given unit vector $\vec{u}$, is it possible to find a vector $\vec{v}$ such that $\vec{u} \cdot \vec{v} = -3$ and $\|\vec{v}\| = 2$? Give an example or explain why this can't be done.

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