Math 2050 Assignment 1, Due September 20, 2017

[3] 1. If $B = (1, -4, 3)$ and $\vec{AB} = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$, find A.

Solution: Let $A = (x, y, z)$. $\vec{AB} = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 1-x \\ -4-y \\ 3-z \end{bmatrix}$. $\Longrightarrow$

$$x = 2, y = -6, z = -2$$

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[4] 2. Given $\vec{u} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, find $\vec{u} - \vec{v}$ and illustrate the subtraction with a picture.

Solution: $\vec{u} - \vec{v} = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$. The picture of the subtraction can be shown by the triangle rule(omitted).

[6] 3. Let $\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\vec{u}_3 = \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$ and $\vec{u}_4 = \begin{bmatrix} 5 \\ 10 \\ 3 \end{bmatrix}$

Are $\vec{u}_1$ and $\vec{u}_2$ parallel? $\vec{u}_1$ and $\vec{u}_3$? $\vec{u}_1$ and $\vec{u}_4$?

Solution: i. $\vec{u_2} = 0\vec{u_1}$. They are parallel.

ii. $\vec{u_3} = 2\vec{u_1}$. They are parallel.

iii. $\vec{u_4}$ is not scalar multiple of $\vec{u_1}$. They are not parallel.

[5] 4. Let $P_1(2, -3, 6)$ and $P_2(2, 2, -4)$. Find the coordinates of the point P on the line segment from P_1 to P_2 such that $\overrightarrow{P_1P} = \frac{2}{5}\overrightarrow{PP_2}$.

Solution: Let the point P have coordinates (x, y, z).

Then, $\overrightarrow{P_1P} = \dfrac{2}{5}\overrightarrow{PP_2}$. This implies that

$\begin{bmatrix} x-2 \\ y+3 \\ z-6 \end{bmatrix} = \dfrac{2}{5} \begin{bmatrix} 2-x \\ 2-y) \\ -4-z \end{bmatrix} \Longleftrightarrow 5 \begin{bmatrix} x-2 \\ y+3 \\ z-6 \end{bmatrix} = 2 \begin{bmatrix} 2-x \\ 2-y \\ -4-z \end{bmatrix}$. Therefore, $5(x-2) = 2(2-x)$, $5(y+3) = 2(2-y)$ and $5(z-6) = 2(-4-z)$. So, $x = 2$, $y = -\frac{11}{7}$ and $z = \frac{22}{7}$

That is, the point P is $P = (2, -\frac{11}{7}, \frac{22}{7})$.

[6] 5. Express each of the following as a single vector.

(a) $3 \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} - 4 \begin{bmatrix} 6 \\ 1 \\ 5 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$ (b) $a \begin{bmatrix} -1 \\ 5 \end{bmatrix} - 3 \begin{bmatrix} -a \\ 2 \end{bmatrix}$

Solution: (a) $\begin{bmatrix} -23 \\ -2 \\ -22 \end{bmatrix}$. (b) $\begin{bmatrix} 2a \\ 5a-6 \end{bmatrix}$.

[6] 6. In each of the following cases, either express $\vec{p}$ as a linear combination of $\vec{u}, \vec{v}, \vec{w}$ or explain why there is no such linear combination.

(a) $\vec{p} = \begin{bmatrix} -4 \\ 7 \\ 5 \\ 0 \end{bmatrix}, \vec{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \vec{v} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \vec{w} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

(b) $\vec{p} = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}, \vec{u} = \begin{bmatrix} 3 \\ 7 \\ 0 \end{bmatrix}, \vec{v} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \vec{w} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

Solution:(a) $\begin{bmatrix} -4 \\ 7 \\ 5 \\ 0 \end{bmatrix} = k_1\vec{u} + k_2\vec{v} + k_3\vec{w}. \quad k_1 = -4, k_2 = 7,\ k_3 = 5.$

(b). $\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} = k_1 \begin{bmatrix} 3 \\ 7 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} + k_3 \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$. This gives

$$\begin{aligned} 3k_1 + 3k_3 &= -1 \\ 7k_1 + 2k_2 + k_3 &= 2 \\ 0 &= 4 \end{aligned}$$

No such k_1, k_2, k_3 exists, implying that $\vec{p}$ is NOT a linear combination of $\vec{u}, \vec{v}$ and $\vec{w}$.

[4] 7. Suppose $\vec{u} = \begin{bmatrix} a \\ -5 \end{bmatrix}, \vec{v} = \begin{bmatrix} 1 \\ 6-b \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$. Find a and b such that $2\vec{u} - 3\vec{v} + 5\vec{w} = \vec{0}$.

Solution:

$$\begin{aligned} 2\vec{u} - 3\vec{v} + 5\vec{w} = \vec{0} &\iff 2\begin{bmatrix} a \\ -5 \end{bmatrix} - 3\begin{bmatrix} 1 \\ 6-b \end{bmatrix} + 5\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ &\iff \begin{bmatrix} 2a - 3 + 15 \\ -10 - 18 + 3b + 10 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ &\iff \begin{bmatrix} 2a + 12 \\ -18 + 3b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ &\iff a = -6 \text{ and } b = 6 \end{aligned}$$

[5] 8. Use vectors to show that the mid-point of the line joining $A(x_1, x_2, x_3)$ to $B(y_1, y_2, y_3)$ is the point $C(\frac{x_1+y_1}{2}, \frac{x_2+y_2}{2}, \frac{x_3+y_3}{2})$.

Solution: Let the point C as (x, y, z). Use the relation $\vec{AC} = \vec{CB}$ to have

$$\begin{bmatrix} x - x_1 \\ y - y_1 \\ z - z_1 \end{bmatrix} = \begin{bmatrix} x_2 - x \\ y_2 - y \\ z_2 - z \end{bmatrix}.$$

This gives $x - x_1 = x_2 - x$, i.e., $x = \frac{x_1+x_2}{2}$. Similarly we have $y = \frac{y_1+y_2}{2}, z = \frac{z_1+z_2}{2}$.

[6] 9. Given three points $A(-1, 0),\ B(2, 3),\ C(4, -1)$ in the plane, find all points D such that ABCD are the vertices of a parallelogram. (hint: three possible locations for D).

Case 1: $\vec{AD_1} = \vec{BC} = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$. $D_1 = (1, -4)$.

Case 2: $\vec{AB} = \vec{CD_2}$. $D_2 = (7, 2)$.

Case 3: $\vec{BC} = \vec{D_3 A}$. $D_3 = (-3, 4)$.

[5] 10. Let $\vec{u} = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 5 \\ 0 \\ -3 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 6 \\ 1 \\ -4 \end{bmatrix}$. Is it possible to find a scalar t such that $\vec{u} + t\vec{v}$ is parallel to $\vec{w}$.

Solution: Suppose $\begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} + t \begin{bmatrix} 5 \\ 0 \\ -3 \end{bmatrix} = k \begin{bmatrix} 6 \\ 1 \\ -4 \end{bmatrix}$. Then, $\begin{bmatrix} -3+5t \\ 2 \\ 1-3t \end{bmatrix} = \begin{bmatrix} 6k \\ k \\ -4k \end{bmatrix}$. $\implies$

(i) $-3 + 5t = 6k$, (ii) $k = 2$ and (iii) $1 - 3t = -4k$.

From (ii) we have, $k = 2$ and so from the (iii) we get $t = 3$.

We need to check if these values of k and t satisfy the first equation.

Substituting in, we get $-3 + 5(3) = 6(2)$ which is correct. So, it is possible to to find such a value for t.

Geometrically, this means that the vectors $\vec{u}, \vec{v}, \vec{w}$ can all lie in the same plane in 3-D space in this particular case which is not true in general in 3-D space.

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