MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 2 **Mathematics 3202** ANSWERS

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Student **Student number**

Marks

[10] 1. Let D be a region in the first quadrant bounded by the curves $x = 0$, $x = y$ and $x^2 + y^2 = 9$.

[1] (a) Sketch the region.

This is the 8th part of a circle of radius 3, center at the origin, between the positive part of y-axis and the diagonal of the first quadrant.

[3] (b) Describe the region in terms of inequalities in Cartesian variables **in two ways**.

The intersection of the circle and the line $x = y$ is $(3/\sqrt{2}, 3/\sqrt{2})$. Thus the two possible descriptions are

$$0 \le x \le 3/\sqrt{2}, \quad x \le y \le \sqrt{9 - x^2}$$

or

$$0 \le y \le 3/\sqrt{2},\ 0 \le x \le y \quad \text{and} \quad 3/\sqrt{2} \le y \le 3,\ 0 \le x \le \sqrt{9 - y^2}$$

[2] (c) Set up integrals for the area of D corresponding to each of the set of inequalities found in (b). Do not evaluate them.

$$\int_0^{3/\sqrt{2}} \int_x^{\sqrt{9-x^2}} \ dydx = \int_0^{3/\sqrt{2}} \int_0^{y} \ dxdy + \int_{3/\sqrt{2}}^{3} \int_0^{\sqrt{9-y^2}} \ dxdy$$

[1] (d) Describe the region in terms of inequalities in the Polar coordinates.

$$0 \le r \le 3, \quad \frac{\pi}{4} \le \theta \le \frac{\pi}{2}.$$

[3] (e) Evaluate $\int\int_D (1 + x^2 + y^2)^{-1}\, dA$ in Polar coordinates.

$$\int_{\pi/4}^{\pi/2} \int_0^3 \frac{r}{1 + r^2}\, drd\theta = \frac{\pi}{8} \ln 10.$$

[10] 2. Consider triple integral describing volume of a solid.

$$V = \int_0^R \int_0^{\sqrt{R^2-x^2}} \int_{-\sqrt{2R^2-x^2-y^2}}^{\sqrt{x^2+y^2}} dzdydx$$

1. Which of the following surfaces form the boundary of the solid?

- cylinder
- plane
- hyperbolic parabolic (saddle)
- elliptic paraboloid
- sphere
- cone

Answer: cylinder (side), two planes(side), cone(top), sphere(botom)

This is a quarter of an ice-cream cone turned upside down.

2. For each integral (a),...,(g) given below identify whether it has the same value as V above or not. (Do not evaluate the integrals).

(a) **Yes**

$$\int_0^R \int_0^{\sqrt{R^2-y^2}} \int_{-\sqrt{2R^2-x^2-y^2}}^{\sqrt{x^2+y^2}} dzdxdy$$

(b) **Yes**

$$\int_0^R \int_0^{\sqrt{R^2-x^2}} \int_{-\sqrt{x^2+y^2}}^{\sqrt{2R^2-x^2-y^2}} dzdydx$$

(c) **No**

$$\int_0^{\pi/2} \int_0^R \int_r^{\sqrt{2R^2-r^2}} r\, dzdrd\theta$$

(d) **Yes**

$$\int_{\pi/2}^{\pi} \int_0^R \int_{-\sqrt{2R^2-r^2}}^{r} r\, dzdrd\theta$$

(e) **No**

$$\int_0^{\pi/2} \int_{\pi/4}^{\pi} \int_0^{\sqrt{2}R} \rho^2 \sin\phi\, d\rho d\phi d\theta$$

(f) **Yes**

$$\int_0^{\pi/2} \int_{\pi/4}^{3\pi/4} \int_0^{\frac{R}{\sin\phi}} \rho^2 \sin\phi\, d\rho d\phi d\theta + \int_0^{\pi/2} \int_{3\pi/4}^{\pi} \int_0^{\sqrt{2}R} \rho^2 \sin\phi\, d\rho d\phi d\theta$$

(g) **No**

$$\int_0^{\pi/2} \int_{\pi/4}^{3\pi/4} \int_0^{R\sin\phi} \rho^2 \sin\phi\, d\rho d\phi d\theta + \int_0^{\pi/2} \int_{3\pi/4}^{\pi} \int_0^{\sqrt{2}R} \rho^2 \sin\phi\, d\rho d\phi d\theta$$

[5] 3. Use Lagrange multipliers method to find maximum value of function $f = x^2y$ along the curve $x^2 + y^2 = 1$. Please, circle correct answer.

$$\sqrt{\frac{2}{27}} \qquad \boldsymbol{\frac{2}{\sqrt{27}}} \qquad \frac{2\sqrt{2}}{3\sqrt{3}} \qquad \frac{2}{3}\sqrt{3}$$

Solve the system

$$2xy = \lambda 2x, \quad x^2 = \lambda 2y, \quad x^2 + y^2 = 1$$

obtain $y = \lambda$ then $x^2 = 2y^2$ so $3y^2 = 1$. Thus $y = \pm 1/\sqrt{3}$, $x^2 = 2/3$.

Thus $f = x^2y = 2/(3\sqrt{3}) = 2/\sqrt{27}$ is the maximum.

Note than there is another solution of the system $x = 0$, $\lambda = 0$, $y = \pm 1$, which gives $f = 0$, so this is not the maximum value.

[5] 4. Evaluate the following integral $\int_C x\,dx + y^2\,dy + z^3\,dz$ along the parametric curve $\vec{r} = (t, \sqrt{t}, t^{1/3})$ from point $(1, 1, 1)$ to point $(64, 8, 4)$

Rewrite

$$x = t, \quad y = \sqrt{t}, \quad z = t^{1/3}, \quad 1 \le t \le 64$$

Thus

$$dx = dt, \quad dy = \frac{1}{2}t^{-1/2}dt, \quad dz = \frac{1}{3}z^{-2/3}dt.$$

Consequently,

$$\int_C x\,dx + y^2\,dy + z^3\,dz = \int_1^{64} (t + \frac{1}{2}t^{1/2} + \frac{1}{3}t^{1/3})dt = \frac{27379}{12}$$

[1] 5. [BONUS] Which of the following is NOT a name of neither a moon of a planet nor an asteroid in Solar system?

Phobos, Deimos, Gaspra, Io, Callisto, Mathilde, Europa, Leda, Atlas, Ida, Janus, Helene, Titan, **Sputnik**, Ophelia, Bianca, Desdemona, Juliet, Eros, Larissa, Triton, Miranda, Luna.