# MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

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TEST 1 **Mathematics 3202** ANSWERS

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**Student** **Student number**

Marks

1. A curve is given by a parametric equation. Name it out of the following list : Ellipse, Parabola, Circle, Hyperbola, Straight line, Helix, Segment of straight line, None of them.

[4] (a) $x = t + 1,\ \ y = 2t + 3,\ \ z = 3t - 7,\ \ -1 \le t \le 1$
Segment of straight line

[4] (b) $x = 3\sqrt{2}\sin(-t),\ \ y = \pi,\ \ z = 5\sqrt{3}\cos(-t),\ \ -\pi \le t \le \pi$
Ellipse.

[4] (c) $x = \tan t,\ \ y = \tan t + 1,\ \ z = 10,\ -\pi/2 < t < \pi/2$
Straight line $y = x + 1$, $-\infty < x < \infty$, $z = 10$.

[4] (d) $x = 0,\ \ \ y = 2\cos t,\ \ z = -2\sin t,\ \ 0 \le t \le 2\pi$
Circle of radius 2.

[6] 2. Let $a > 0$ and $b > 0$ be positive constants, and the position vector be given as

$$\vec{r} = (at; -b\sin 2t; -b\cos 2t).$$

Which of the following gives the distance traveled over time interval $0 \le t \le 1$?

Please circle correct answer.

$a^2+4b^2$ $\quad 2(a^2+b^2)^{1/2}$ $\quad (a^2-4b^2)^{1/2}$ $\quad 2(a^2+2b^2)^{1/2}$ $\quad (a^2+4b^2)^{1/2}$ $\quad 2(a^2/4+b^2)^{3/2}$.

$$\int_0^1 \sqrt{a^2 + 4b^2(\cos^2(2t) + \sin^2(2t))}\, dt = (a^2 + 4b^2)^{1/2}$$

[6] 3. Let $a > 0$ and $b > 0$ be positive constants. Consider the parametric curve $\vec{r} = (a, b\cos 3t, b\sin 3t)$. The curvature of this curve at time $t = \pi/12$ is [please, choose a correct answer]

$$\frac{3}{ab} \qquad \frac{1}{b} \qquad \frac{\sqrt{2}}{2b} \qquad \frac{1}{a} \qquad \frac{3}{a^2+b^2} \qquad a \qquad b$$

The curve is a circle of radius $b$. Thus the curvature is $1/b$.

Unletnatively, this answer can be obtained using the formula $\kappa = |\vec{v} \times \vec{a}|v^{-3}$, where $\vec{v} = (0, -3b\sin 3t, 3b\cos 3t)$, $v = 3b$, and $\vec{a} = (0, -9b\cos 3t, -9\sin 3t)$. But this is evidently longer solution.

4. Given the position vector $\vec{r} = (e^t, 2e^t \sin t, 2e^t \cos t)$

[5] (a) Show that speed is $v = 3e^t$

$$\vec{v} = (e^t, 2e^t(\sin t + \cos t), 2e^t(-\sin t + \cos t))$$

then

$$v = \sqrt{e^{2t} 4((\sin t + \cos t)^2 + (-\sin t + \cos t)^2)} = 3e^t.$$

[5] (b) Find the unit tangent vector $\vec{T}(t)$.

$$\vec{T}(t) = \frac{\vec{v}}{v} = \frac{1}{3}(1, 2(\sin t + \cos t), 2(-\sin t + \cos t))$$

[5] (c) Find the unit normal vector $\vec{N}(t)$. First, find a normal vector

$$\frac{d\vec{T}}{dt} = \frac{1}{3}(0, 2(\cos t - \sin t), 2(-\cos t - \sin t)),$$

its length is $\sqrt{8}/3$. Thus the unit normal vector

$$\vec{N}(t) = \frac{1}{\sqrt{8}}(0, 2(\cos t - \sin t), 2(-\cos t - \sin t)).$$

[5] (d) Show that $\vec{T}(t)$ and $\vec{N}(t)$ are orthogonal for all $t$.

$$\vec{T}(t) \cdot \vec{N}(t) = 0.$$

[6] 5. Find the gradient vector for $f(x, y) = (15x^2 + 12y^2)^{1/3}$ at point $(1; 1)$.

$$\frac{\partial f}{\partial x} = 10x(15x^2 + 12y^2)^{-2/3} = 10/9, \quad \frac{\partial f}{\partial y} = 8y(15x^2 + 12y^2)^{-2/3} = 8/9,$$

[12] 6. Let $f(4;5) = 6$ and the gradient vector of $f(x, y)$ at the point $(4; 5)$ be $\nabla f(4; 5) = (2; 3)$. Which of the following are NOT equations of the plane tangent to the surface $z = f(x, y)$ at the point $(4, 5, 6)$.

(a) $2(x - 4) + 3(y - 5) - (z - 6) = 0$

(b) $-20(x - 4) - 30(y - 5) + 10(z - 6) = 0$

(c) $2(x - 4) + 3(y - 5) + (z - 6) = 0$ This is not

(d) $2(x - 2) + 3(y - 5) - (z - 6) = 4$

(e) $z = 2x + 3y - 17$

(f) $z = 20x + 30y - 170$ This is not

[9] 7. [BONUS] Which of the following are names of the moons of planets in Solar system?

Warning: You will get +0.5 point for each correct answer and -0.5 for each incorrect.

Phobos (Mars),

Deimos (Mars),

Io (Jupiter),

Callisto (Jupiter),

Europa (Jupiter),

Leda (Jupiter),

Atlas (Saturn),

Janus (Saturn),

Helene (Saturn),

Titan (Saturn),

Ophelia (Uranus),

Bianca (Uranus),

Desdemona (Uranus),

Juliet (Uranus),

Larissa (Neptune),

Triton (Neptune),

Miranda (Uranus),

Luna (= the Moon, Earth).

Answer: They all are!