MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

FINAL EXAM **Mathematics 3202** FALL 2003

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Marks

[25] 1. The velocity of a charged particle moving in a magnetic field and subject to the Lorentz force is the vector function

$$\mathbf{v}(t) = \langle \sin t,\ \cos t,\ 3 \rangle.$$

The initial position of the particle is $\mathbf{r}(0) = \langle 0, 0, -3 \rangle$.

[2] (a) Find the acceleration of the particle as a vector-function of t.

[5] (b) Find the position as a vector-function of t. Can you identify the shape of the trajectory?

[4] (c) Find the distance traveled from $t = 0$ until $t = 10$.

[5] (d) Find an equation of the normal plane to the trajectory at the initial position.

[9] (e) Find an equation of the osculating plane to the trajectory at the initial position.

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[10] 2. For the given function

$$F(x, y) = \frac{y^2}{2x + y^2}$$

[5] (a) Give a parametric description of a curve along which $\lim\limits_{(x,y)\to(0,0)} F(x, y) = -1.$ in

[5] (b) Is the function $F(x, y)$ continuous at the point $(0, 0)$? Explain your answer.

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[10] 3.

[5] (a) Find point(s) on the surface $z = y^2 - x^2 - 1$ where the normal line is parallel to the vector $\langle -1, 1, 1 \rangle$.

[5] (b) Find the equation of the tangent plane to the surface at the point where $x = 1$, $y = 2$.

[6] 4. Find the rate of change of the function $F(x, y) = \sinh(xy)$ at the point $(\ln 2, 1)$ in the direction $\theta = \pi/3$.

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[14] 5. Consider the vector field $\vec{F}(x, y) = (2x - y,\ 2y - x)$.

[4] (a) Show that it is concervative.

[5] (b) Find the potential function $f(x, y)$.

[5] (c) Evaluate (by any means) the line integral of $\vec{F}(x, y)$ along the segment of parabola $y = x^2$, $x \in [-1, 2]$.

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[8] 6. Evaluate by reversing the order of integration

$$\int_0^1 \int_x^1 e^{-y^2}\, dy\, dx.$$

[9] 7. Evaluate the line integral $\oint_C y dx - x dy$, where the curve C is the circle $x^2 - 2x + y^2 = 3$ in two ways:

[5] (a) directly

[4] (b) using Green's theorem

[6] 8. Find the volume of the solid tower with square base $0 \leq x \leq 1,\ 0 \leq y \leq 1$ at the level $z = 0$ and the roof described by the equation $z = \sqrt{xy}$.

[12] 9. Solve **ONE** of the following problems. (If you solve more than one, you will receive the mark for the best out of 3 solutions, plus bonus 50% of the mark for the second best solution plus 25% of the mark for the third one.)

[12] (a) Use the Lagrange Multipliers Method to find maximum of the function $F(x, y) = x^2 y$ with $x \geq 0, y \geq 0$, subject to the constraint $x + y = 1$.

[12] (b) Verify Stokes' theorem for the vector field $\vec{F} = \langle 3y, 2z, x\rangle$ and the surface S which is the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$.

[12] (c) Use the Divergence theorem to evaluate the flux across the surface of the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 2)$ of the vector field $\vec{F} = \langle xy, y^2, x^2\rangle$.