Math 3202 **Answers** **Assignment #7**

1. Evaluate the line integral along given curve by two methods: (a) directly (b) using Green's Therem

   (a) $\oint_C xy^2\,dx + x^3\,dy$, where C is a rectangle with vertices $(0,0)$ $(2,0)$ $(2,3)$ $(0,3)$.
   Solution
   (a) Line integral along each side of the rectangle gives $0 + 24 - 18 + 0 = 6$.
   (b) by Green's Theorem, $\int_0^2 \int_0^3 (3x^2 - 2xy)\,dydx = 6$.

   (b) $\oint_C x\,dx + y\,dy$, where C consists of the line segments from $(0,1)$ to $(0,0)$ from $(0,0)$ to $(1,0)$ and the parabola $y = 1 - x^2$ from $(1,0)$ to $(0,1)$.
   Solution
   (a) Line integral along each side of the region gives $\int_0^1 (0dt + (1-t)(-dt)) + \int_0^1 (tdt + 0dt) + \int_0^1 ((1-t)dt + (2t - t^2)(2-2t))dt = 0$.
   (b) by Green's Theorem, $\int\int_D 0\,dA = 0$.

   (c) $\oint_C e^y\,dx + 2xe^y\,dy$, where C is square with sides $x = 0$ $x = 1$ $y = 0$ $y = 1$.
   Solution:
   (a) line integrals give $1 + (2e - 2) - e + 0 = e - 1$
   (b) by Green theorem $\int_0^1 \int_0^1 (2e^y - e^y)dxdy = e - 1$

   (d) $\oint_C x^2y^2\,dx + 4xy^3\,dy$, where C is a triangle with vertices $(0,0)$ $(1,3)$ $(0,3)$.
   Solution:
   (a) line integrals give $333/5 - 3 + 0 = 318/5$
   (b) by Green theorem $\int_0^3 \int_0^{y/3} (4y^3 - 2x^2y)dxdy = 318/5$

2. Evaluate the line integral using Green's Theorem.

   (a) $\oint_C \sin y\,dx + x\cos y\,dy$, where C is the ellipse $x^2 + xy + y^2 = 1$.
   Solution:
   $$\oint_C \sin y\,dx + x\cos y\,dy = \int\int_D (\cos y - \cos y)\,dA = 0$$

   (b) $\oint_C e^x + x^2y\,dx + e^y - xy^2\,dy$, where C is the circle $x^2 + y^2 = 25$, oriented cklockwise.
   Solution:
   $$\oint_C e^x + x^2y\,dx + e^y - xy^2\,dy = -\int\int_D (-y^2 - x^2)\,dA = \int\int_D (y^2 + x^2)\,dA = \int_0^5 \int_0^{2\pi} r^2\,r\,d\theta dr = \frac{625}{2}\pi$$

3. Find the work done by the force $\vec{F} = (x(x+y), xy^2)$ in moving a particle from the origin along the line segment to $(1,0)$ then along the line segment to $(0,1)$ and then back to the origin along the y-axis.

   Solution: The work is represented by line integral $\int_C \vec{F}d\vec{r} = \int_C x(x+y)dx + xy^2dy =$, where the path C is the triangle with vertices at (0,0), (1,0) and (0,1). By Green's theorem this line integral is equal to
   $$\int_0^1 \int_0^{1-x} (y^2 - x)\,dydx = -\frac{1}{12}$$

4. Find the curl of vector field

$$\vec{F} = \left(\frac{x}{x^2+y^2+z^2}, \frac{y}{x^2+y^2+z^2}, \frac{z}{x^2+y^2+z^2} \right)$$

Solution:

$$\mathrm{curl}\vec{\mathrm{F}} = \left[\frac{-2\mathrm{yz}+2\mathrm{yz}}{(\mathrm{x}^2+\mathrm{y}^2+\mathrm{z}^2)^2}, \frac{-2\mathrm{xz}+2\mathrm{xz}}{(\mathrm{x}^2+\mathrm{y}^2+\mathrm{z}^2)^2}, \frac{-2\mathrm{yx}+2\mathrm{yx}}{(\mathrm{x}^2+\mathrm{y}^2+\mathrm{z}^2)^2} \right] = [0,0,0] = \vec{0}.$$