Math 3202 **Answers** **Assignment #1**

1. Given parametric equations find equation in the form $F(x, y) = 0$ and sketch the curve. Do you know its name?

   (a) $x = \sec t$, $y = \tan t$, $-\pi/2 < t < \pi/2$
   Answer: $1 - y^2 - x^2 = 0$, $x > 0$, the right branch of hyperbola with vertex at (1,0) and asymptotes $y = \pm x$.
   use $1 + \tan^2 t = \sec^2 t$, and $\sec t > 0$ for $-\pi/2 < t < \pi/2$.

   (b) $x = e^t$, $y = 1 - e^{-t}$, $-\ln 2 \le t \le \ln 2$
   Answer: $1 - 1/x - y = 0$, $x > 0$, the right branch of hyperbola with vertex at at (1,0) and asymptotes $y = 1$, $x = 0$.

   (c) $x = 2\cos t$, $y = 3\sin t$, $-\pi < t < \pi$
   Answer: $x^2/4 + y^2/9 - 1 = 0$, ellipse.

   (d) $x = 2\cos(2t)$, $y = 2\sin(2t)$, $-\pi/2 < t < \pi/2$
   Answer: $x^2/4 + y^2/4 - 1 = 0$, circle of radius 2.

2. For each curve from the previous problem find and sketch tangent vector at $t = 0$.

   a) *Answer:* $\vec{r}'(0) = (0, 1)$

   b) *Answer:* $\vec{r}'(0) = (1, 1)$

   c) *Answer:* $\vec{r}'(0) = (0, 3)$

   d) *Answer:* $\vec{r}'(0) = (0, 4)$

   P.S. Note that in each case $\vec{r}'(0)$ is tangent to a corresponding curve at point $\vec{r}(0)$.

3. Sketch 3D curve and find tangent vector at each time t.

   (a) $x = \cos(2t)$, $y = 1$, $z = 2$, $-\pi/2 < t < \pi/2$
   Answer: segment of straight line parallel to the x-axis and connecting poins (-1,1,2) and (1,1,2).

   (b) $x = \cos(2t)$, $y = 3$, $z = \sin(2t)$, $-\pi/2 < t < \pi/2$
   Answer: circle in the plain $y = 3$, radius =1, center at $(0, 3, 0)$.

   (c) $x = \cos(2t)$, $y = t$, $z = \sin(2t)$, $-\pi/2 < t < \pi/2$
   Answer: Part of helix along the y-axis.

4. Find parametric equations for the sides of the triangle with vertices at points $(1, 2, 3)$, $(-1, 5, 6)$, $(0, 5, 1)$.

   Answer: Side 1: $x = t - (1 - t) = 2t - 1$, $y = 2t + 5(1 - t) = -3t + 5$, $z = 3t + 6(1 - t) = -3t + 6$, $0 \le t \le 1$.

   Side 2: $x = -t$, $y = 5t + 5(1 - t) = 5$, $z = 6t + (1 - t) = 5t + 1$, $0 \le t \le 1$.

   Side 3: $x = (1 - t)$, $y = 5t + 2(1 - t) = 3t + 2$, $z = t + 3(1 - t) = -2t + 3$, $0 \le t \le 1$.

5. Find parametric equations for the curve of intersection of cylinder $x^2 + y^2 = 4$ and $z = xy$. Can you imagine and sketch the curve? Please!

   Answer: $x = 2\cos t$, $y = 2\sin t$, $z = 2\sin(2t)$.

   This curve is an intersection of the circular cyllinder and the seddle shaped surface.

6. Find vector position vector $\vec{r}(t)$ if the velocity vector is given by

   (a) $\dfrac{4}{1+t^2}\mathbf{i} + \dfrac{2t}{1+t^2}\mathbf{j} + \dfrac{t^2}{1+t^2}\,\mathbf{k}$

   Answer: $\vec{r} = (4\arctan t)\mathbf{i} + \ln(1+t^2)\mathbf{j} + (t - \arctan t)\mathbf{k}$

   (b) $e^{2t}\,\mathbf{i} + \ln t\,\mathbf{j} + \sec^2 t\,\mathbf{k}$

   Answer:$\vec{r} = (e^{2t}/2)\mathbf{i} + (t\ln t - t)\mathbf{j} + \tan t\mathbf{k}$.