Math 3202 **Due Thur March 23** **Assignment #7**

1. Evaluate the surface integral
   (a) $\int\int_{\mathbf{S}} xy\,dS$, where $\mathbf{S}$ is the triangular region with vertices (1,0,0,), (0,2,0), and (0,0,2).
   (b) $\int\int_{\mathbf{S}} x^2z^2\,dS$, where $\mathbf{S}$ is a part of the cone $z^2 = x^2 + y^2$ that lies between the planes $z = 1$ and $z = 3$.
   (c) $\int\int_{\mathbf{S}}(x^2y + z^2)\,dS$, where $\mathbf{S}$ is a part of the cylinder $x^2 + y^2 = 9$ between the planes $z = 0$ and $z = 2$.
   (d) $\int\int_{\mathbf{S}}(x^2 + y^2 + z^2)\,dS$, where $\mathbf{S}$ is a part of the cylinder $x^2 + y^2 = 9$ between the planes $z = 0$ and $z = 2$ together with its top and bottom disks.

2. Evaluate the surface integral $\int\int_{\mathbf{S}} \vec{F} \cdot d\vec{S}$ for the given vector field $\vec{F}$ and the oriented surface $\mathbf{S}$.
   (a) $\vec{F}(x, y, z) = (xy, yz, zx)$, $\mathbf{S}$ is the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the square $0 \le x \le 1$, $0 \le y \le 1$, and has upward orientation.
   (b) $\vec{F}(x, y, z) = (xy, 4x^2, yz)$, $\mathbf{S}$ is a surface $z = xe^y$ that lies above the square $0 \le x \le 1$, $0 \le y \le 1$, and has upward orientation.
   (c) $\vec{F}(x, y, z) = (xze^y, -xze^y, z)$, $\mathbf{S}$ is the part of the plane $x + y + z = 1$ in the first octant and downward orientation.
   (d) $\vec{F}(x, y, z) = (x, y, z^4)$, $\mathbf{S}$ is the part of the cone $z = \sqrt{x^2 + y^2}$ beneath the plane $z = 1$ with downward orientation.
   (e) $\vec{F}(x, y, z) = (x, -z, y)$, $\mathbf{S}$ is the part of the sphere $x^2 + y^2 + z^2 = 4$ in the first octant, with orientation toward the origin.

3. A fluid with density 1200 flows with velocity $\vec{v} = (y, 1, z)$. Find the rate of flow upward through the paraboloid $z = 9 - (x^2 + y^2)/4$, $x^2 + y^2 \le 36$.

4. Use Stokes' Th to evaluate $\int_C \vec{F} \cdot d\vec{r}$. Curve C is oriented counterclockwise as viewed from above.
   (a) $\vec{F}(x, y, z) = (x + y^2, y + z^2, z + x^2)$, C is the triangle with vertices (1,0,0), (0,1,0), and (0,0,1).
   (b) $\vec{F}(x, y, z) = (e^{-x}, e^x, e^z)$, C is the boundary of the plane $2x + y + 2z = 2$ in the first octant.
   (c) $\vec{F}(x, y, z) = (yz, 2xz, e^{xy})$, C is the circle $x^2 + y^2 = 16$, $z = 5$.

5. (bonus) If $\mathbf{S}$ is a sphere and $\vec{F}$ satisfies the hypotheses of the Stokes' Th, show that $\int\int_{\mathbf{S}} \vec{F} \cdot d\vec{S} = 0$