Answers

This is just answers. Solutions will be distributed in class.

- 1. Evaluate the line integral by two methods: directly and using Green's Th
 - (a) $\int_C y \, dx x \, dy$, where C is a circle of radius R. Answer: $-2\pi R^2$.
 - (b) $\int_C xy \, dx + x^2 y^3 \, dy$, where C is a triangle with vertices (0,0), (1,0), and (1,2). Answer: 2/3.
- 2. Use Green's Th to evaluate the line integral along the given positively oriented curve.
 - (a) $\int_C (y + e^{\sqrt{x}}) dx + (2x + \cos y^2) dy$, where C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$. Answer:1/3
 - (b) $\int_C xe^{-2x} dx + (x^4 + 2x^2y^2) dy$, where C is the boundary of the region between two circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. Answer: 0
 - (c) $\int_C y^2 \cos x \, dx + (x^2 + 2y \sin x) \, dy$, where C is the triangle from (0,0) to (2,6) to (2,0) to (0,0). Answer: -16
- 3. Find the work done by force field $\vec{F} = (x; x^3 + 3xy^2)$ moving an object from (-2,0) to (2,0) along x-axis and then along the semicircle $y = (4 x^2)^{1/2}$ to the starting point. (Use Green's Th) Answer: 12π
- 4. Find parametric representation for the surface.
 - (a) The lower part of the ellipsoid $2x^2 + 4y^2 + z^2 = 1$ Answer: $\vec{r}(u, v) = (u, v, -\sqrt{1 - 2u^2 - 4v^2})$, where $2u^2 + 4v^2 \le 1$
 - (b) The part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the cone $z = (x^2 + y^2)^{1/2}$. Answer: $\vec{r}(u, v) = (u, v, \sqrt{4 - u^2 - v^2})$, where $u^2 + v^2 \leq 2$
 - (c) The part of the the cone $z^2 = x^2 + y^2$ that lies inside the sphere $x^2 + y^2 + z^2 = 4$. Answer: $\vec{r}(u, v) = (u \cos v, u \sin v, u), |u| \le \sqrt{2}, 0 \le v \le 2\pi$
 - (d) The part of the cylinder $x^2 + y^2 = 16$ that lies between z = 2 and z = -2. Answer: $\vec{r}(u, v) = (4 \cos v, 4 \sin v, u), |u| \le 2, 0 \le v \le 2\pi$
- 5. Find the area of the surface.
 - (a) The part of the surface $y = 4x + z^2$ that lies between planes x = 0, x = 1, z = 0, z = 1. Answer $\int_0^1 \int_0^1 \sqrt{17 + 4z^2} dx \, dz = \frac{\sqrt{21}}{2} + \frac{17}{4} \left(\ln(2 + \sqrt{21}) - \ln\sqrt{17} \right)$
 - (b) The helicoid $\vec{r}(u,v) = (u\cos v, u\sin v, v), \ 0 \le u \le 1, \ 0 \le v \le \pi.$ Answer $\int_0^{\pi} \int_0^1 \sqrt{1+u^2} du \, dv = \frac{\pi}{2} \left(\sqrt{2} + \ln(1+\sqrt{2})\right)$