Math 3202 **Answers** **Assignment #3**

1. The position function of the spaceship is

$$\vec{r}(t) = (\cos t, \sin t, \tan t)$$

and the coordinates of the space station are $\vec{R} = (-\sqrt{2},\ 2\sqrt{2},\ 7)$.

a) At what moment of time t should the captain turn off the engines in order to coast into the station?

Solution. Set up equations $\vec{r}(t_0) + k\vec{v}(t_0) = \vec{R}$. Solve to get $t_0 = \pi/4$, $k = 3$.

The captain should turn off the engines at time $t = \pi/4$.

b) What is the angular momentum $\vec{L}$ of the spaceship at the moment of time when the engine was turned off. (Recall $\vec{L} = \vec{r} \times \vec{v}$, assuming that mass is 1.)

Answer. $(\sqrt{2}/2,\ -3\sqrt{2}/2,\ 1)$.

c) Find the speed and the distance traveled by the spaceship with the engine turned off.

Solution.

The speed is $|\vec{v}(\pi/4)| = \sqrt{5}$ miles/min.

The distance betweet points $\vec{r}(\pi/4)$ and $\vec{R}$ is $3\sqrt{5}$ miles.

d) How long it takes the spaceship to reach the station after turning off the engine?

Answer. 3 minutes.

(you may assume that time is measured in minutes and distance in miles.)

2. Evaluate the line integral along given curve

(a) $\int ye^x\,ds$, along the line segment jointing $(1,2)$ to $(4,7)$.
Answer.
$= \int_0^1 (2+5t)e^{1+3t}\sqrt{34}dt = \sqrt{34}e(16e^3 - 1)/9$.

(b) $\int (2x+9z)\,ds$, along the arc $x = t$, $y = t^2$, $z = t^3$, $0 \le t \le 1$.
Answer.
$= \int_0^1 (2t+9t^3)\sqrt{1+4t^2+9t^4}dt = (14^{3/2} - 1)/6$.

(c) $\int x^2 z\,ds$, along the line segment jointing $(0,6,-1)$ to $(4,1,5)$.
Answer.
$= \int_0^1 (4t)^2(6t-1)\sqrt{77}dt = 56\sqrt{77}/3$.

3. Find the mass and the center of mass of a thin wire in the shape of

a) quarter-circle $x^2 + y^2 = 4$, $x \ge 0$, $y \ge 0$, if the density function is $\rho(x,y) = x + y$.

Answer.

$$m = \int_0^{\pi/2} (2\cos t + 2\sin t)2dt = 8.$$

$$\bar{x} = \frac{1}{m}\int_0^{\pi/2} 2\cos t(2\cos t + 2\sin t)2dt = \frac{\pi+2}{4}.$$

$$\bar{y} = \frac{1}{m}\int_0^{\pi/2} 2\sin t(2\cos t + 2\sin t)2dt = \frac{\pi+2}{4}.$$

b) helix $x = t$, $y = \cos t$, $z = \sin t$, $0 \le t \le 2\pi$ if the density at any point is equal to the square of the distance from the origin.

Answer.

$$m = \int_0^{2\pi} (t^2+1)\sqrt{2}dt = 2\sqrt{2}\pi\frac{3+4\pi^2}{3}.$$

$$\bar{x} = \frac{1}{m}\int_0^{2\pi} t(t^2+1)\sqrt{2}dt = \frac{3\pi(2\pi^2+1)}{3+4\pi^2}.$$

$$\bar{y} = \frac{1}{m}\int_0^{2\pi} \cos t(t^2+1)\sqrt{2}dt = \frac{6}{3+4\pi^2}.$$

$$\bar{z} = \frac{1}{m}\int_0^{2\pi} \sin t(t^2+1)\sqrt{2}dt = -\frac{6\pi}{3+4\pi^2}.$$

4. Answer questions 1 – 3 for each of the following functions (a) –(f).
   1. Sketch level curves $f(x,y) = k$, $k = 0, 1, 2$ (if such a curve exists).
   2. Name and sketch the surface given by $z = f(x,y)$.
   3. Find partial derivatives f_x and f_y.

(a) $f(x,y) = \sqrt{25 - 4x^2 - y^2}$,

Answer.

1. Level curves are ellipses with equations $4x^2 + y^2 = 25 - k^2$, k=0,1,2.
2. $z \ge 0$, $4x^2 + y^2 + z^2 = 25$ upper half of ellipsoid.
3. $f_x = -4x(25 - 4x^2 - y^2)^{-1/2}$, $f_y = -y(25 - 4x^2 - y^2)^{-1/2}$.

(b) $f(x,y) = \sqrt{x^2 + \frac{y^2}{9} - 1}$,

1. Level curves are ellipses with equations $x^2 + y^2/9 = 1 + k^2$, k=0,1,2.
2. $z \ge 0$, $x^2 + y^2/9 - z^2 = 1$ upper half of hyperboloid of one sheet.
3. $f_x = x(x^2 + y^2/9 - 1)^{-1/2}$, $f_y = (y/9)(x^2 + y^2/9 - 1)^{-1/2}$.

(c) $f(x,y) = \sqrt{2 + x^2 + \frac{y^2}{4}}$,

1. k=0,1 no level curves; k=2 ellipse $x^2/2 + y^2/8 = 1$.
2. $z \ge 0$, $-x^2 - y^2/4 + z^2 = 2$ upper half of hyperboloid of two sheet.
3. $f_x = x(2 + x^2 + y^2/4)^{-1/2}$, $f_y = (y/4)(2 + x^2 + y^2/4)^{-1/2}$.

(d) $f(x,y) = x^2 - \frac{y^2}{4}$,

1. k=0 lines $y = \pm 2x$; k=1,2 hyperbolas
2. $z = x^2 - y^2/4$ hyperbolic paraboloid.

3 $f_x = 2x$, $f_y = -y/2$.

(e) $f(x,y) = 5 - x^2 - \frac{y^2}{4}$,

1. Level curves are ellipses with equations $x^2 + y^2/4 = 5 - k$, k=0,1,2.
2. $z = 5 - x^2 - y^2/4$ elliptic paraboloid. (up side down, lifted up by 5 units.)
3 $f_x = -2x$, $f_y = -y/2$.

(f) $f(x,y) = \sqrt{y^2 + \frac{x^2}{4}}$,

1. Level curves are ellipses with equations $x^2/4 + y^2 = k^2$, k=1,2; for k=0 just a point at the origin.
2. $z \geq 0$ $z^2 = x^2/4 + y^2$ upper part of cone.
3 $f_x = (x/4)(y^2 + \frac{x^2}{4})^{-1/2}$, $f_y = y(y^2 + \frac{x^2}{4})^{-1/2}$.