

1. Show that if a particle moves with a constant speed, then the velocity and acceleration vectors are orthogonal.
2. Use definition of the cross product and the product rule for differentiation to show that

$$(\vec{a} \times \vec{b})' = \vec{a}' \times \vec{b} + \vec{a} \times \vec{b}'.$$

3. Find the tangential and normal components of the acceleration vector if the position is given by
 - a) $\vec{r} = (3t - t^3; 3t^2; 0)$
 - b) $\vec{r} = (e^t; \sqrt{2}t; e^{-t})$
4. Find the tangential, normal, and binormal vectors for curve $\vec{r}(t) = (e^t, e^t \sin t, e^t \cos t)$ at point $(1, 0, 1)$.
5. Find equations of normal plane and osculating plane for the curve $\vec{r} = (t; t^2; t^3)$ at point $(1, 1, 1)$.
6. Find osculating circle of the ellipse $9x^2 + 4y^2 = 36$ at points $(2, 0)$ and $(0, 3)$.
7. Derive the formula for the arc length

$$l = \int_0^T \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} dt$$

for an arc given by $\vec{r}(t) = (x(t); y(t); z(t))$, $t \in [0, T]$.