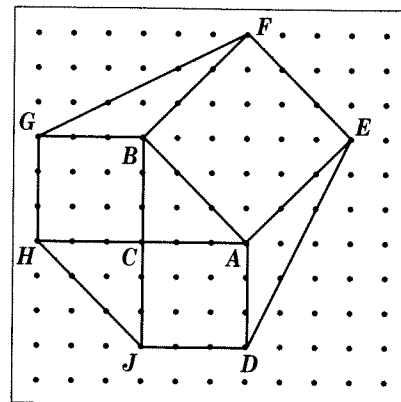
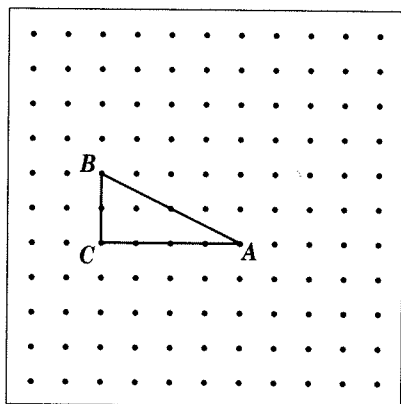


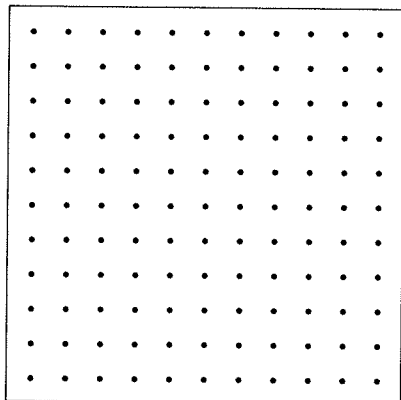
## MAKING A TRIANGLE CONJECTURE

1. Starting with right isosceles triangle  $ABC$  shown at right, squares are drawn on each of its sides. Triangles  $HCJ$ ,  $GBF$ , and  $DAE$  are drawn as shown. Find the areas of the four triangles and enter the numbers in the chart on this sheet.



2. Initial triangle  $ABC$  shown at left is a right triangle. Draw the square on each side of triangle  $ABC$  and complete the three triangles as in question 1. Be sure to label the vertices as before. Find the areas of the four triangles and enter the numbers in the chart on this sheet.

3. Draw your own example below. Starting with a *right triangle*  $ABC$ , draw the squares on each of its sides and complete the other three triangles as before. Find the areas of the four triangles and enter the numbers in the chart on this sheet.



| Question | Area of triangle |       |       |       |
|----------|------------------|-------|-------|-------|
|          | $ABC$            | $HCJ$ | $GBF$ | $DAE$ |
| 1        | 4.5              |       |       |       |
| 2        | 4                |       |       |       |
| 3        |                  |       |       |       |

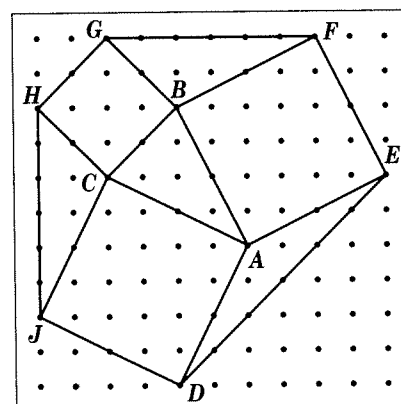
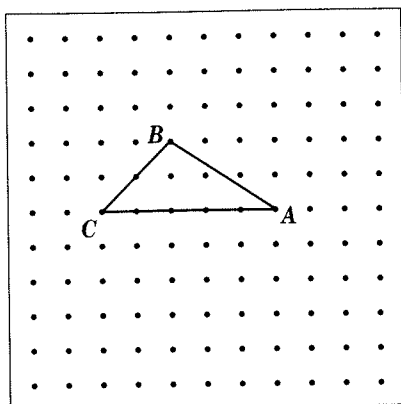
Refer to the table to answer these questions.

- What relationship do you see among the areas of the four triangles in question 1? \_\_\_\_\_
- Does the same relationship hold for the four triangles in question 2? \_\_\_\_\_
- Does the relationship also hold for the four triangles in question 3? \_\_\_\_\_
- Will your conclusion always be true if you start with *any* right triangle? \_\_\_\_\_

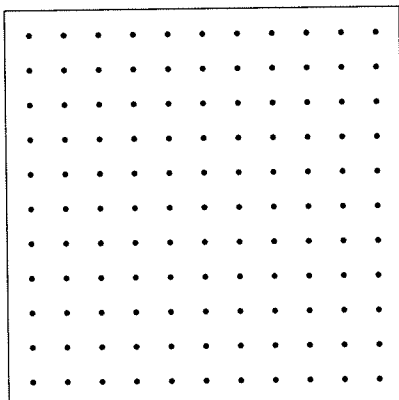
Note: If you are not sure about your conclusion, use dot paper to draw some more triangles with squares on each side, as in question 1, and find the areas of the four triangles.

- Complete the following conjecture that is based on the results of the chart. If squares are drawn on each side of a right triangle and the three triangles are completed as shown in question 1, then the area of each of the three triangles is equal to \_\_\_\_\_.

1. In these figures, the initial triangle  $ABC$  is not a *right* triangle. Starting with isosceles triangle  $ABC$  shown at the right, squares are drawn on each of its sides. Triangles  $HCJ$ ,  $GBF$ , and  $DAE$  are drawn as shown. Find the areas of the four triangles and enter the numbers in the chart on this sheet.



2. Initial triangle  $ABC$  shown to the left is a scalene triangle. Draw the square on each of its sides and complete the triangles as in question 1. Be sure to label the vertices as before. Find the areas of the four triangles and enter the numbers in the chart on this sheet.



| Question | Area of triangle |       |       |       |
|----------|------------------|-------|-------|-------|
|          | $ABC$            | $HCJ$ | $GBF$ | $DAE$ |
| 1        | 6                |       |       |       |
| 2        | 5                |       |       |       |
| 3        |                  |       |       |       |

Refer to the table to answer these questions.

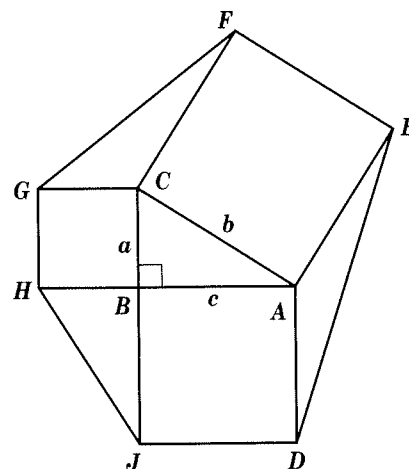
- What relationship do you see among the areas of the four triangles in question 1? \_\_\_\_\_
  - Does the same relationship hold for the four triangles in question 2? \_\_\_\_\_
  - Does the relationship also hold for the four triangles in question 3? \_\_\_\_\_
  - Will your conclusion always be true if you start with *any* triangle? \_\_\_\_\_
- Note: If you are not sure about your conclusion, use dot paper to draw some more triangles with squares on each side, as in question 1, and find the areas of the four triangles.
8. Complete the following conjecture that is based on the results of the chart. If squares are drawn on each side of a triangle and the three triangles are completed as shown in question 1, then the area of each of the three triangles is equal to \_\_\_\_\_.

CONJECTURE 1. *If squares are drawn on each side of a right triangle ABC and the three triangles are completed as shown, then the area of each of the three triangles is equal to the area of triangle ABC.*

*Proof.* Write out your proof (or describe your method of proof) below.

Now that you have proved your conjecture, you can restate it as a theorem:

THEOREM 1. *If squares are drawn on each side of a right triangle ABC and the three triangles are completed as shown, then the area of each of the three triangles is equal to the area of triangle ABC.*

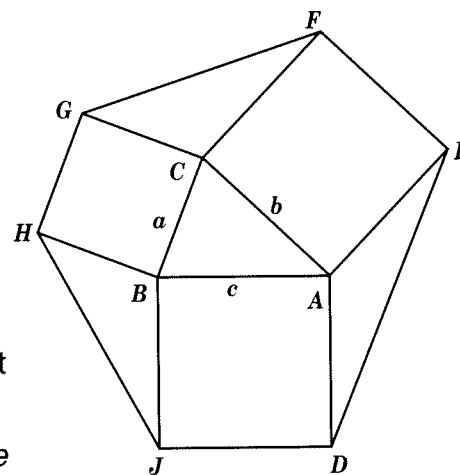


CONJECTURE 2. *If squares are drawn on each side of any triangle ABC and the three triangles are completed as shown, then the area of each of the three triangles is equal to the area of triangle ABC.*

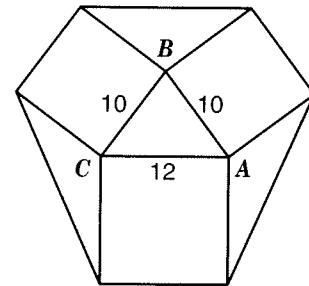
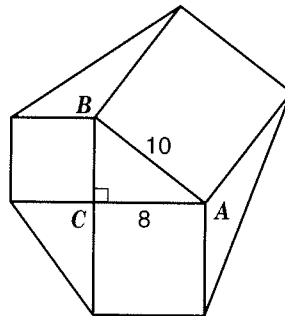
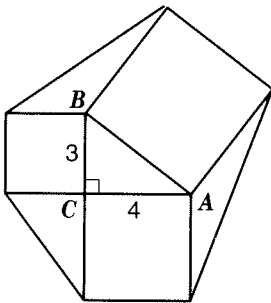
*Proof.* Write out your proof (or describe your method of proof) below.

Now that you have proved your conjecture, you can restate it as a theorem:

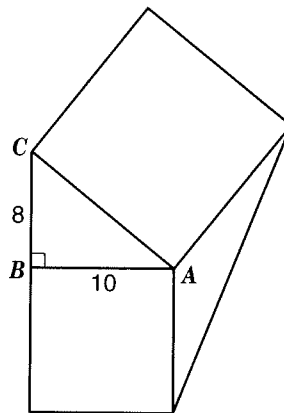
THEOREM 2: *If squares are drawn on each side of any triangle ABC and the three triangles are completed as shown, then the area of each of the three triangles is equal to the area of triangle ABC.*



1. Squares are drawn on each side of any triangle  $ABC$  as shown. Find the area of each triangle.



2. Squares are drawn on sides  $AB$  and  $AC$  of right triangle  $ABC$  shown below. Find the area of each triangle.



3. Squares are drawn on sides  $AC$  and  $BC$  of right triangle  $ABC$  shown below. Find the area of each triangle.

